Cooperative Diversity Routing in Wireless Networks

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***Abstract*—In this paper, we explore physical layer *cooperative communication* in order to design network layer *routing* algorithms that are energy efficient. We assume each node in the network is equipped with a single omnidirectional antenna and that multiple nodes are able to coordinate their transmissions in order to take advantage of spatial diversity to save energy. Specifically, we consider *cooperative diversity* at physical layer and *multi-hop routing* at network layer, and formulate *minimum energy routing* as a *joint* optimization of the transmission power at the physical layer and the link selection at the network layer. We then show that as the network becomes larger, finding optimal cooperative routes becomes computationally intractable. As such, we develop a number of heuristic routing algorithms that have polynomial computational complexity, and yet achieve significant energy savings. Simulation results are also presented, which indicate that the proposed algorithms based on optimal power allocation significantly outperform existing algorithms based on equal power allocation, by more than 60% in some simulated scenarios.**

***Index Terms*—Minimum energy routing, cooperative communication, cooperative diversity, wireless networks.**

I. Introduction

Energy efficiency is a challenging problem in wireless networks, especially in ad hoc and sensor networks, where network nodes are typically battery powered. It is not therefore surprising that energy efficient communication in wireless networks has received significant attention in the past several years. Most of the work in this area has specifically focused on designing energy efficient network and physical layer mechanisms. At the network layer, the goal is to find energy efficient *routes* that minimize transmission power in an end-to-end setting. At the physical layer, the goal is to design energy efficient communication schemes for the wireless medium. One such scheme is the so-called *cooperative communication* [1], [2].

Most routing protocols for ad hoc networks consider a network as a graph of point-to-point links, and multiple links are used to transmit data from a source node to a destination node in a multi-hop fashion. Although the notion of a link has been a useful abstraction for wired networks, for wireless networks, the notion of a link is vague [2]. Wireless networks, however, are often constrained by the same notion of link that is inherited from wired networks, namely, concurrent transmissions of multiple nearby transmitters result in interference producing a collision. Cooperative communication is a radically different paradigm in which the conventional notion of a link is abandoned. Specifically, some of the constraints imposed by the conventional definition of a link are violated, *e.g.*, a link can originate from multiple transmitters, and concurrent transmissions, when coordinated, do not result in collision [2]. To this end, we note that multi-hop communication in wireless networks is a special case of cooperative communication.

Although there has been considerable research on energy efficient routing (*e.g.*, [3]), and cooperative communication (*e.g.*, [4]), in isolation, only recently a few works have addressed network layer routing and physical layer cooperation problems *jointly* [5]–[7]. This is surprising as cooperative communication is inherently a network solution; hence, it is essential to investigate routing and cooperation jointly. This is the problem we address in this paper for cooperative wireless networks. Our objective is to *find routes that are energy efficient while guaranteeing some minimum end-to-end throughput*.

The existing literature in this area can be divided into two categories, as follows. The first category assumes a *static* environment in which sets of transmitting nodes are phase-locked and perfect channel state information is available; in this case, nodes are capable of cooperatively *beamforming* to a receiver. A notable example is the work presented in [5] (and its subsequent extensions such as [8]), where optimal power allocation and routing are formulated. Whereas there have been recent examples of cooperative beamforming [9], the synchronization requirements for such are onerous in a mobile ad hoc network, and thus we turn to the second category. In the second category, routing decisions and cooperative transmission are performed without channel state information. The work presented in [6] is an example in this category, where a set of *adjacent* nodes cooperatively transmit to a receiver with *equal* transmission power.

Whereas we argue that the first category (*i.e.*, cooperative beamforming) faces significant implementation challenges, we argue that current solutions in the second category (*i.e.*, equal power allocation) are far from being optimal. In this work, we assume that only the fading *distribution* is known at the transmitters, and *jointly* formulate *optimal power allocation* and *cooperative routing*. In particular, we consider a general cooperation scheme in which multiple transmitters cooperatively send data to multiple receivers. However, because of the inherent difficulties and inefficiency in performing distributed receiver cooperation, receivers individually receive and decode transmitted data. Receivers that are successful in such decoding can then join the transmitting set.

Our contributions can be summarized as follows:

1) We formulate energy optimal cooperative routing subject to constraints on individual node transmission power and achievable end-to-end throughput.
2) We formulate optimal power allocation for a cooperative link between a set of transmitters and a set of receivers assuming only statistical knowledge about the fading process.
3) We develop optimal and heuristic cooperative routing algorithms, and evaluate their performance using simulations.

The rest of this paper is organized as follows. In Section II, we describe the system model considered in this paper, and formulate cooperative link cost in terms of transmission power. Section III presents our formulation of optimal cooperative routing, and describes a few heuristic routing algorithms to avoid the complexity of optimal routing. Simulation results are presented in Section IV, where we compare the energy cost of different cooperative routing algorithms. Finally, our conclusions as well as future research directions are discussed in Section V.

II. System Model

We consider a wireless network consisting of a set of nodes distributed randomly in an area, where each node has a single omnidirectional antenna. We assume that each node can adjust its transmission power and that multiple nodes can coordinate their transmissions at the physical layer to form a cooperative link. For the latter, recall that only rough packet synchronization is required [4].

A. Channel Model

The channel between each pair of transmitting and receiving nodes is a time-slotted wireless channel. Consider a transmitting set $T = \{t_1, \ldots, t_m\}$ and a receiving set $R = \{r_1, \ldots, r_n\}$ forming a cooperative link. Let $x_i[t]$ and $y_j[t]$ denote transmitted and received signals in time-slot t at nodes $t_i \in T$ and $r_j \in R$, respectively. Without loss of generality, we assume that $x_i[t]$ has unit power and that transmitter t_i is able to control its power $p_i[t]$ in arbitrarily small steps up to some limit $\mathrm{P}_{\max}$. Let $\eta_j[t]$ denote the noise and other interferences received at r_j, where $\eta_j[t]$ is assumed to be additive white Gaussian with power density P_{n_j}. For notational simplicity, we omit the time-slot index t throughout the paper. The model for the discrete-time received signal at each node r_j is then expressed as follows

$$y_j = \sum_{t_i \in T} \sqrt{\frac{p_i}{d_{ij}^{\alpha}}} h_{ij}\, x_i + \eta_j, \tag{1}$$

where, d_{ij} is the distance between nodes t_i and r_j, α is the path-loss exponent, h_{ij} is the complex channel gain between t_i and r_j modeled as $h_{ij} = |h_{ij}| e^{j\theta_{ij}}$, where $|h_{ij}|$ is the channel gain magnitude and θ_{ij} is the phase. Using this model, the received power at node r_j is given by the following relation

$$p_j = \sum_{t_i \in T} \left(\frac{|h_{ij}|^2}{d_{ij}^{\alpha}} \right) p_i\,. \tag{2}$$

Finally, every node has a limit on its maximum transmission power denoted by $\mathrm{P}_{\max}$.

B. Cooperation Model

Per Section I, cooperation at a given stage consists of a collection of multiple-input single-output (MISO) links, where a set of transmitters T cooperatively send data to a set of receivers R. Since we do not consider receiver cooperation, each receiver has to individually receive and decode the data. We assume a non line-of-sight (LOS) environment, implying that $|h_{ij}|$ has a Rayleigh distribution (which is widely used in literature [10]) with unit variance, *i.e.*, $\mathbb{E}\left[|h_{ij}|^2\right] = 1$.

Let $\mathbf{P}$ denote the set of all *feasible* power allocation vectors $\mathbf{p}$, where $\mathbf{p}_i$ is the power allocated to transmitter $t_i \in T$. We have

$$\mathbf{P} = \{\mathbf{p} |\, \mathbf{p}_i \leq \mathrm{P}_{\max}\}, \tag{3}$$

where, $\mathrm{P}_{\max}$ is the maximum transmission power of a transmitter. Let γ_{ij} denote the Signal-to-Noise-Ratio (SNR) at receiver $r_j \in R$ due to transmitter $t_i \in T$. It is obtained that

$$\gamma_{ij} = \frac{1}{d_{ij}^{\alpha}} \frac{\mathbf{p}_i}{\mathrm{P}_{n_j}} |h_{ij}|^2, \tag{4}$$

where, P_{n_j} is the noise power at receiver r_j. Since $|h_{ij}|$ is Rayleigh distributed with unit variance, $|h_{ij}|^2$ is exponentially distributed with mean 1. Consequently, γ_{ij} is exponentially distributed with mean

$$\bar{\gamma}_{ij} = \frac{1}{d_{ij}^{\alpha}} \frac{\mathbf{p}_i}{\mathrm{P}_{n_j}}\,. \tag{5}$$

Let γ_j denote the total SNR due to m transmitters at receiver r_j. We have

$$\gamma_j = \sum_{i=1}^{m} \gamma_{ij}, \tag{6}$$

which is the summation of m independent and exponentially distributed random variables γ_{ij}. Then, the probability density function of γ_j denoted by $f_{\gamma_j}(.)$ can be expressed as

$$f_{\gamma_j}(y) = \sum_{i=1}^{m} \frac{\Pi_{ij}}{\bar{\gamma}_{ij}} e^{-y/\bar{\gamma}_{ij}}, \tag{7}$$

where,

$$\Pi_{ij} = \prod_{\substack{k=1 \\ k \neq i}}^{m} \frac{\bar{\gamma}_{ij}}{\bar{\gamma}_{ij} - \bar{\gamma}_{kj}}\,. \tag{8}$$

To derive the above expressions, consider the case of having only two transmitters, *i.e.*, $m = 2$. We have $\gamma_j = \gamma_{1j} + \gamma_{2j}$. Therefore,

$$f_{\gamma_j}(y) = f_{\gamma_{1j}} * f_{\gamma_{2j}}(y),$$

which is the convolution of $f_{\gamma_{1j}}$ and $f_{\gamma_{2j}}$. It is obtained that

$$f_{\gamma_j}(y) = \frac{1}{\bar{\gamma}_{1j} - \bar{\gamma}_{2j}} \left(e^{-y/\bar{\gamma}_{1j}} - e^{-y/\bar{\gamma}_{2j}} \right), \\ = \frac{e^{-y/\bar{\gamma}_{1j}}}{\bar{\gamma}_{1j} - \bar{\gamma}_{2j}} + \frac{e^{-y/\bar{\gamma}_{2j}}}{\bar{\gamma}_{2j} - \bar{\gamma}_{1j}} .$$

After computing $f_{\gamma_j}(y)$ for a few values of m, the general form of (7) emerges. An alternative approach for deriving the distribution of the sum of independent exponential random variables based on hypoexponential distribution is presented in the Appendix.

The cooperative link from T to R consists of n MISO channels. For the MISO channel that reaches receiver r_j (referred to as MISO channel j throughout the paper), the instantaneous channel capacity under power allocation $\mathbf{p}$ is given by (see [10])

$$c_j(\mathbf{p}) = \log_2 (1 + \gamma_j) . \tag{9}$$

In our cooperation model, every transmitter t_i transmits data at rate λ that is fixed across the transmitters. Ideally, every receiver r_j should receive data at the rate λ as well. However, due to fading, the corresponding MISO channel may not be able to sustain the rate λ resulting in *outage*. Let $\wp_j(\mathbf{p}, \lambda)$ denote the probability that the MISO channel j is in outage for power allocation $\mathbf{p}$ and transmission rate λ. We obtain that:

$$\begin{aligned} \wp_j(\mathbf{p}, \lambda) &= \mathbb{P}\{c_j(\mathbf{p}) < \lambda\} \\ &= \mathbb{P}\left\{\gamma_j < 2^{\lambda} - 1\right\} . \end{aligned} \tag{10}$$

Let $\mathrm{SNR}_{\min}$ denote the minimum SNR required to achieve rate λ, that is $\mathrm{SNR}_{\min} = 2^{\lambda} - 1$. Then, $\wp_j(\mathbf{p}, \lambda)$ can be computed as follows:

$$\begin{aligned} \wp_j(\mathbf{p}, \lambda) &= \mathbb{P}\{\gamma_j < \mathrm{SNR}_{\min}\} \\ &= \int_0^{\mathrm{SNR}_{\min}} \sum_{i=1}^{m} \frac{\Pi_{ij}}{\bar{\gamma}_{ij}} e^{-y/\bar{\gamma}_{ij}} dy \\ &= \sum_{i=1}^{m} \Pi_{ij} (1 - e^{-\mathrm{SNR}_{\min}/\bar{\gamma}_{ij}}) . \end{aligned} \tag{11}$$

C. Routing Model

A K-hop cooperative path ℓ is a sequence of K cooperative links $\{\ell_1, \ldots, \ell_K\}$, where link ℓ_k is formed between a set of transmitters T_k and a set of receivers R_k using cooperative transmission at the physical layer. The sequence of links ℓ_k connects a source 's' to a destination 'd' in a loop-free path. Our objective is to find a path that minimizes end-to-end transmission power to reach the destination subject to a constraint on the throughput[1] of the path. Let $\mathcal{C}(T_k, R_k)$ denote the *cost* of link ℓ_k, which is defined as the minimum transmission power to form cooperative link ℓ_k, *i.e.*, the minimum total power to reach R_k from T_k in a single-hop cooperative transmission. The problem of energy efficient routing can be formulated as follows

$$\begin{aligned} &\min_{\ell} \sum_{\ell_k \in \ell} \mathcal{C}(T_k, R_k) \\ &s.t. \quad \rho(\ell) \geq \rho_0, \end{aligned} \tag{12}$$

where, $\rho(\ell)$ is the end-to-end throughput of path ℓ, and ρ_0 is a target throughput. Let $\rho(\ell_k)$ denote the throughput of link $\ell_k \in \ell$ (note the slight abuse of the notation). Then $\rho(\ell)$ can be expressed as

$$\rho(\ell) = \min_{\ell_k \in \ell} \rho(\ell_k) . \tag{13}$$

Since throughput is an increasing function of the transmission power, a necessary condition for minimizing power over a path ℓ is given by $\rho(\ell_k) = \rho_0$, for all $\ell_k \in \ell$, *i.e.*, all links should just achieve the minimum throughput ρ_0.

For the future references, Table I summarizes the notation that is used throughout this paper. Some notations will be precisely defined in the next sections, where we discuss different aspects of our routing algorithm.

TABLE I
NOTATION USED IN THE PAPER.

Notation	Description
$\mathrm{P}_{\max}$	Maximum transmission power.
$\mathbf{p}$	Power allocation vector.
$\mathbf{p}_i$	Power allocated to transmitter t_i.
$\mathbf{P}$	Set of all feasible power allocation vectors.
d_{ij}	Distance between nodes t_i and r_j.
h_{ij}	Complex channel gain between t_i and r_j.
γ_{ij}	Exponential random variable denoting SNR at receiver r_j due to transmitting node t_i.
$\bar{\gamma}_{ij}$	Mean for random variable γ_{ij}.
γ_j	Hypoexponential random variable denoting SNR at receiver r_j.
$\wp_j(\mathbf{p}, \lambda)$	Outage probability for power allocation $\mathbf{p}$, and transmission rate λ at receiver r_j.
$\rho(\ell)$	End-to-end throughput of path ℓ.
$\rho(\ell_k)$	Throughput of link ℓ_k.
$\rho_j(\mathbf{p}, \lambda)$	Throughput of MISO channel j subject to power allocation $\mathbf{p}$, and transmission rate λ.
$\mathcal{C}(T, R)$	Transmission cost for the cooperative link ℓ_{TR}.
$\mathcal{R}(T, R)$	Remaining cost of reaching the destination, if R is chosen as the receiving set.
$\mathcal{P}(T)$	Total transmission cost to reach the destination from transmitting set T.

III. COOPERATIVE ROUTE SELECTION

In this section, we first formulate the transmission cost for cooperative communication between two sets of nodes. We then develop optimal and heuristic algorithms to find energy efficient cooperative routes in an arbitrary wireless network.

[1]We define *throughput* as the long-term average error-free rate at which data is transmitted, aka *goodput*.

A. Link Cost Formulation

Consider a cooperative link ℓ_{TR} that is formed between the transmitting set $T = \{t_1, \ldots, t_m\}$ and the receiving set $R = \{r_1, \ldots, r_n\}$. Such a link is composed of n MISO channels corresponding to the n receivers. Recall that we defined $\mathcal{C}(T,R)$ as the minimum transmission power to form a cooperative link between T and R. Our objective here is to compute $\mathcal{C}(T,R)$ subject to a target throughput ρ_0 over the corresponding cooperative link ℓ_{TR}.

Let $\rho_j(\mathbf{p},\lambda)$ denote the throughput of MISO channel j subject to power allocation $\mathbf{p}$ and transmission rate λ. We obtain that

$$\rho_j(\mathbf{p},\lambda) = \lambda(1 - \wp_j(\mathbf{p},\lambda))\,. \tag{14}$$

It is clear now that different MISO channels can support different throughputs. In theory, *multiple description coding* [11] can be used to allow receivers to receive data at potentially different rates, hence achieving different throughputs over different MISO channels. However, in this work, for the ease of exposition, we restrict the discussion to the case where all receivers receive the same data at the same rate, and leave the exploration of different receiving rates to a future work. In this case, the transmission rate λ is chosen so that the slowest channel can achieve the throughput ρ_0. Therefore, for a given $\mathbf{p}$ and λ, the link throughput $\rho(\ell_{TR})$ is given by

$$\rho(\ell_{TR}) = \min_{r_j \in R} \rho_j(\mathbf{p},\lambda)\,. \tag{15}$$

Therefore, the link cost $\mathcal{C}(T,R)$ for the *cooperative link* ℓ_{TR} is formulated as the following optimization problem:

$$\begin{aligned} \mathcal{C}(T,R) =& \min_{\mathbf{p}\in\mathbf{P}} \sum_{t_i \in T} \mathbf{p}_i \\ & s.t. \quad \exists \lambda > 0 : \min_{r_j \in R} \rho_j(\mathbf{p},\lambda) = \rho_0\,. \end{aligned} \tag{16}$$

This optimization problem can be solved numerically, as shown in Section IV. Let $\mathbf{p}^*_{TR}$ and λ^*_{TR} denote, respectively, the optimal power allocation vector and transmission rate computed in (16).

B. Optimal Link Selection

At each step of routing (corresponding to a *hop*), the routing algorithm should choose R from all the nodes that have not received the data yet so that the end-to-end power consumption is minimized. To this end, we design a routing algorithm that generalizes the classical Bellman-Ford algorithm to handle a set of receivers as opposed to a single receiver. Let $\mathcal{P}(T)$ denote the total transmission power to reach the destination from transmitting set T using multi-hop cooperative transmissions. Then, R is implicitly given by the following optimization problem

$$\mathcal{P}(T) = \min_{R \subseteq \overline{T}} \{\mathcal{C}(T,R) + \mathcal{R}(T,R)\}\,, \tag{17}$$

where, $\mathcal{R}(T,R)$ denotes the remaining cost of reaching the destination if R is chosen as the receiving set, and $\overline{T}$ denotes the set of potential receivers, *i.e.*, nodes that are not in T. After the transmission, every $r_j \in R$ that is not in outage will be added to the transmitting set for the next hop. Therefore, we obtain that

$$\begin{aligned} \mathcal{R}(T,R) &= \sum_{R_{\text{out}} \subseteq R} \Big[\mathcal{P}(T \cup \overline{R}_{\text{out}} |\ R_{\text{out}} \text{ in outage}) \\ &\qquad\qquad \times \mathbb{P}\{R_{\text{out}} \text{ in outage}\}\Big] \\ &= \sum_{R_{\text{out}} \subseteq R} \Big[\mathcal{P}(T \cup \overline{R}_{\text{out}}) \\ &\qquad\qquad \times \prod_{r_j \in R_{\text{out}}} \wp_j(\mathbf{p}^*_{TR}, \lambda^*_{TR}) \\ &\qquad\qquad \times \prod_{r_i \in \overline{R}_{\text{out}}} (1 - \wp_i(\mathbf{p}^*_{TR}, \lambda^*_{TR}))\Big], \end{aligned} \tag{18}$$

where, R_{out} denotes the set of receivers that are in outage, and $\overline{R}_{\text{out}} = R \setminus R_{\text{out}}$, *i.e.*, the set of receivers that are not in outage.

C. Cooperative Routing Algorithm

An iterative implementation of the routing algorithm works in rounds. Let h denote the round number, and augment all routing related variables with h, *e.g.*, $\mathcal{P}^h(T)$ denotes the routing cost from T to the destination in round h. Routing variables are updated in each round as follows

$$\mathcal{P}^{h+1}(T) = \min_{R \subseteq \overline{T}} \left\{\mathcal{C}(T,R) + \mathcal{R}^h(T,R)\right\}\,, \tag{19}$$

where, $\mathcal{R}^h(T,R)$ is computed based on $\mathcal{P}^h(T)$ using (18). The algorithm terminates when

$$\mathcal{P}^{h+1}(T) = \mathcal{P}^h(T), \quad \text{for all } T \subseteq \mathcal{N}, \tag{20}$$

where, $\mathcal{N}$ is the set of all network nodes. Initially, the only potential transmitter is the source node, *i.e.*, $T = \{s\}$. To initialize the routing variables, we take

$$\mathcal{P}^0(T) = \infty, \quad \text{for all } T \subseteq \mathcal{N} \tag{21}$$

$$\mathcal{P}^h(T) = 0, \quad \text{if } d \in T \text{ for all } T \subseteq \mathcal{N} \tag{22}$$

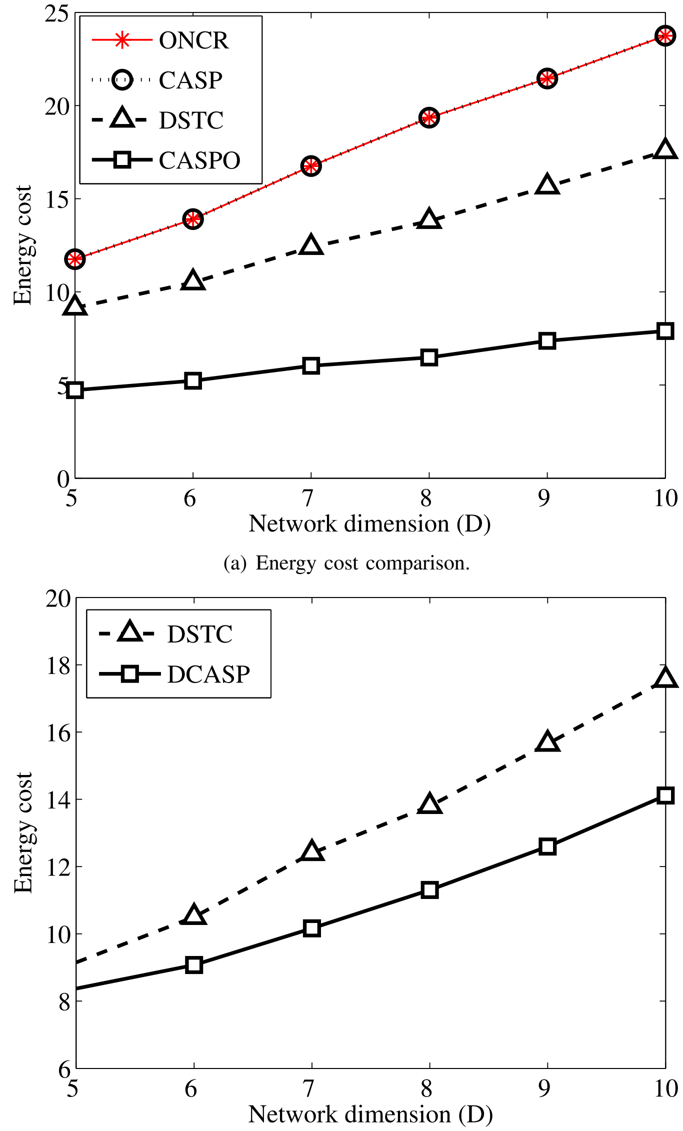
where, 'd' denotes the destination node.

D. Heuristic Cooperative Routing

Ideally, in each step of the routing algorithm, we should identify a *set* of receivers, *i.e.*, R, and then solve the power allocation problem (formulated in (16)) simultaneously for all the receivers. Such an approach however is computationally expensive. Solving the minimization problem (19), in each round of the algorithm, involves enumeration of $O(2^N)$ subsets (where, $N = |\mathcal{N}|$). There are $O(2^N)$ sets T in the network as well, and hence, $O(2^N)$ rounds for the algorithm to converge (see the convergence condition in (20)). However, if we restrict R to sets of size K_1 then the complexity of each round is reduced to solving the power allocation problem for $O(N^{K_1})$ subsets. Similarly, if we restrict T to subsets of size K_2, then the number of rounds is reduced to $O(N^{K_2})$. Thus, the routing complexity will become polynomial in the network size N for the restricted transmitter/receiver case.

In this subsection, we propose a number of heuristic algorithms that while having a lower computational complexity compared to the optimal routing algorithm, still achieve significant energy savings, as will be shown in Section IV.

1) **Cooperation Along the Shortest Path (CASP)**
In every step of the cooperative routing, the next node along the *non-cooperative shortest path* is selected as the receiving node. After the transmission, if the receiving node is not in outage, it will be added to the transmitting set for the next step of the routing.
2) **Opportunistic Cooperation Along the Shortest Path (CASPO)**
This algorithm is similar to CASP with the addition of *overhearing*. After the transmission to the next node along the shortest non-cooperative path, all the nodes that are not in outage will be added to the transmitting set for the next step of the routing.
3) **K-Transmitter Cooperation Along the Shortest Path (KT-CASPO)**
This algorithm is a variation of CASPO, in which the transmitting set consists of only the closest K transmitters to the receiver.
4) **K-Receiver Cooperation Along the Shortest Path (KR-CASPO)**
The number of receivers at each step of routing is limited to K nodes. The K nodes consist of the next node on the non-cooperative shortest path together with the $(K-1)$-nearest neighbors of that node.
5) **K-Receiver Optimal Cooperation (K-OPT)**
In every step of the routing algorithm, the optimal receiving set of size K or smaller is selected. The routes computed using this approach are not necessarily optimal as the receiving set is limited to K-node or smaller subsets only. Comparing 1-OPT against CASP, however, provides some insight about the optimaility/efficiency of the wildly used cooperation along the shortest non-cooperative path algorithms (for example, see [5] and [7]).



(a) Energy cost comparison.

(b) Optimal power allocation.

Fig. 1. Energy cost of different routing algorithms.

IV. Performance Evaluation

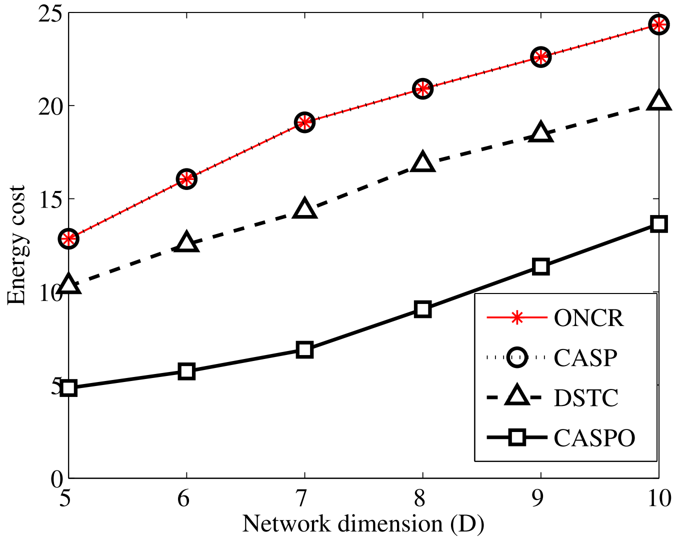
We have simulated the routing algorithms discussed in the previous section to evaluate their performance numerically in some sample networks. In the following subsections, we present our simulation results and compare the performance of different algorithms in terms of energy consumption.

A. Simulation Parameters

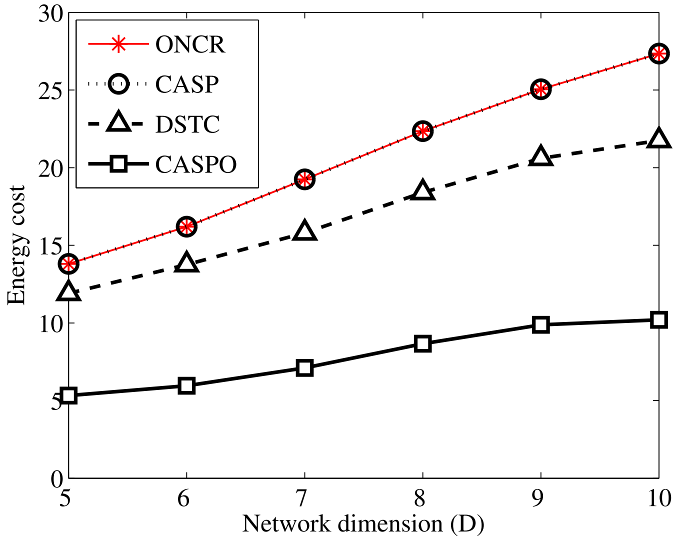
We simulate a wireless network, in which nodes are deployed uniformly at random. The network coverage forms a square of area $D \times D$, and node density is set to 2, *i.e.*, there are $N = 2D^2$ nodes in the network. We choose two nodes s and d located at the lower left and the upper right corners of the network, respectively, and find cooperative and non-cooperative routes from s to d. We then compute the total amount of energy consumed on each route using different routing algorithms. For simulation purposes, we take $\mathrm{P}_{\max} = 1$, $\alpha = 2$ and $\mathrm{P}_{n_j} = 1$ for every node j. In the implementation of all the algorithms, a fixed throughput $\rho_0 = 0.2$ has been considered so that the only measure for comparison is the energy consumption. The total energy consumption for each case is obtained by averaging over 20 simulation runs with different seeds.

In the simulations, in addition to the algorithms described in Section III, we implement the following algorithms:

1) **Optimal Non-Cooperative Routing (ONCR)**
This is the least-cost non-cooperative route computed using Dijkstra's algorithm.
2) **Distributed Spatio-Temporal Cooperation (DSTC)**
This is the equal power allocation cooperative routing algorithm proposed in [6].

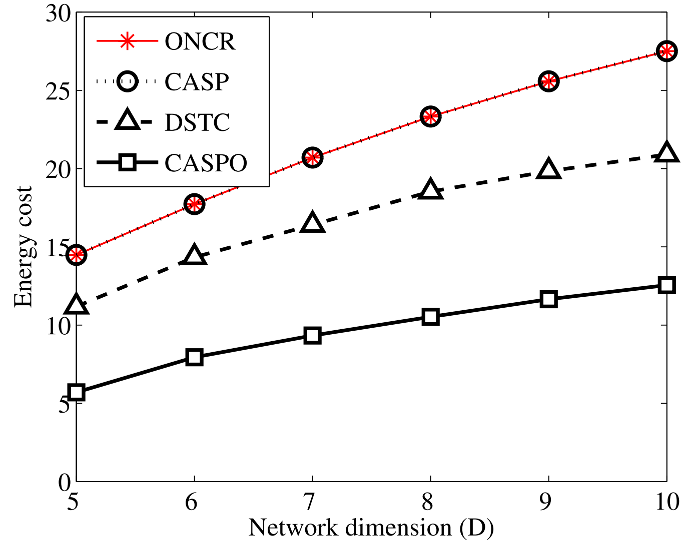


(a) Path-loss exponent (α) = 3.

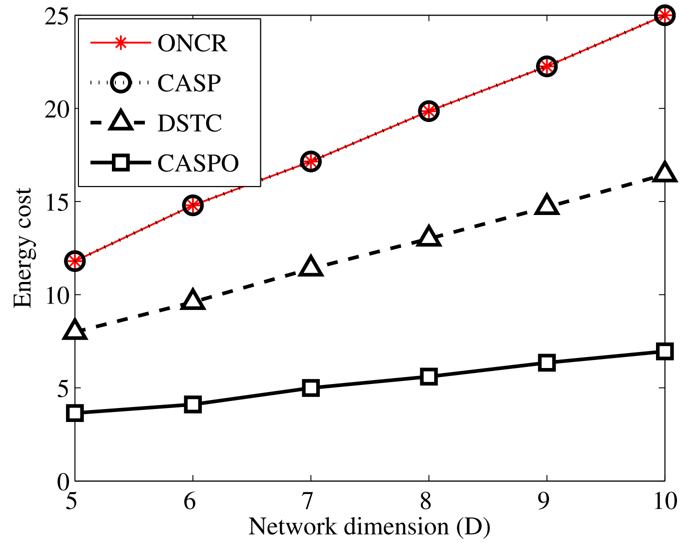


(b) Path-loss exponent (α) = 4.

Fig. 2. Effect of path loss.



(a) Node density = 1.



(b) Node density = 3.

Fig. 3. Effect of node density.

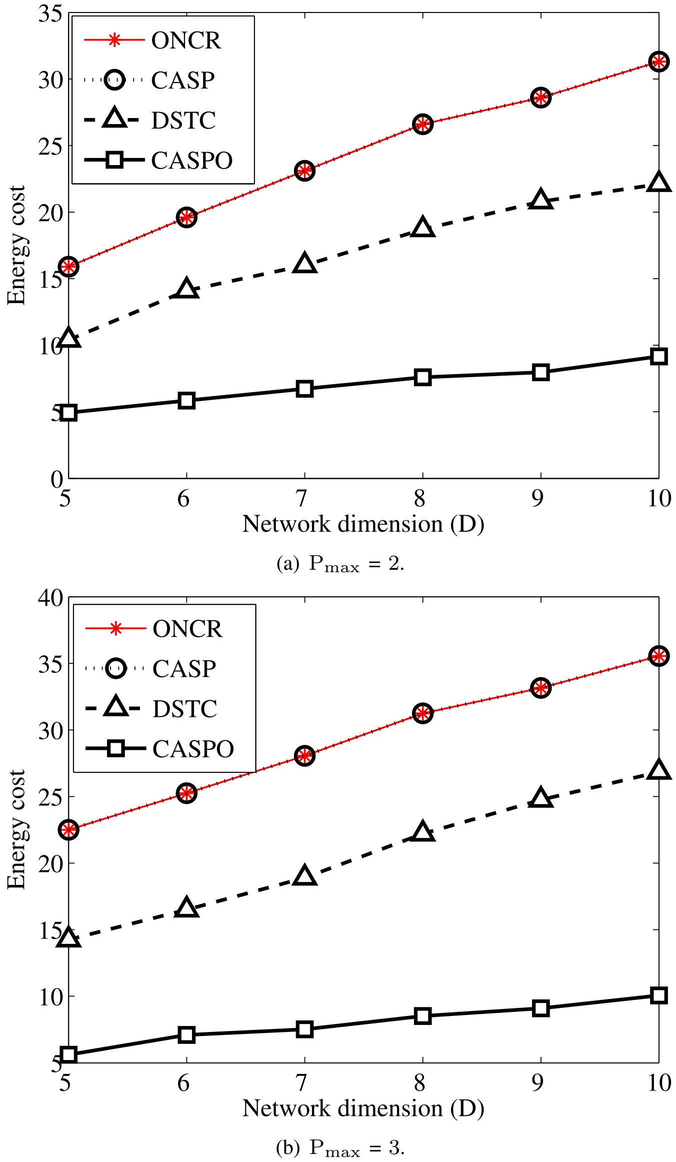
B. Simulation Results

In this subsection, we present our simulation results and study the impact of different network parameters on the performance of the proposed cooperative routing algorithm.

1) Optimal Power Allocation: Fig. 1(a) summarizes the main result of this paper, which shows that optimal power allocation combined with opportunistic route selection, as done in CASPO, achieve significant energy savings, outperforming equal power allocation (*i.e.*, DSTC) by more than 60%. We also observe that CASP, surprisingly, performs just like the non-cooperative algorithm. The reason is that, in simulated topologies, the distance between the successive transmitters is so large that essentially power is allocated only to the transmitter that is closest to the next node along the shortest path, *i.e.*, no gain is obtained from transmitter diversity.

To isolate the effect of power allocation and compare optimal and equal power allocation schemes, we have implemented a modified version of the CASP algorithm called *Distributed CASP (DCASP)*. In DCASP, transmitting and receiving sets are chosen according to DSTC, but the transmission power is allocated optimally using (16). Fig. 1(b) compares the performance of DCASP and DSTC. It is observed that DCASP achieves about 20% energy savings compared to DSTC, in the simulated scenarios, indicating that equal power allocation (*e.g.*, [6] and [12]) is *not* able to fully exploit cooperative diversity.

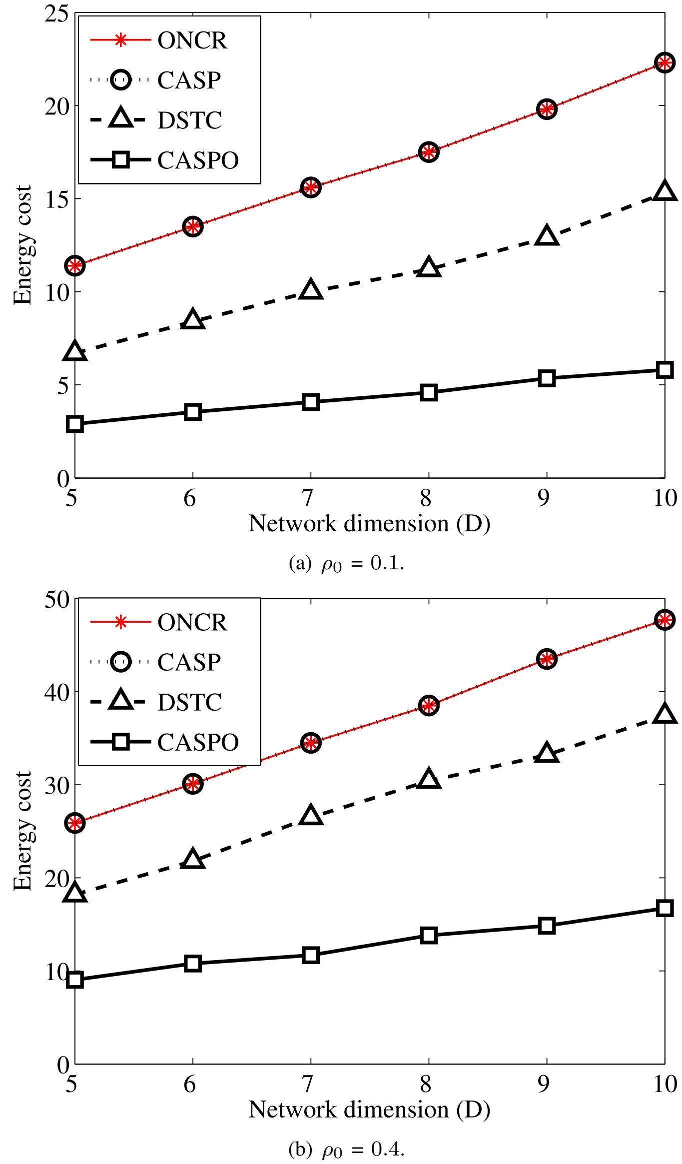
2) Effect of Path-Loss: The effect of *path-loss* exponent (α) on energy cost of different routing algorithms is presented in Figs. 2(a) and 2(b). Although path-loss affects the energy cost of different algorithms, the overall performance behavior does not change with respect to α. Specifically, CASPO achieves the lowest energy cost among the simulated algorithms.



(a) $\mathrm{P_{max}}$ = 2.

(b) $\mathrm{P_{max}}$ = 3.

Fig. 4. Effect of transmission power ($\mathrm{P_{max}}$).



(a) ρ_0 = 0.1.

(b) ρ_0 = 0.4.

Fig. 5. Effect of path throughput (ρ_0).

3) Effect of Node Density: Fig. 3 shows the impact of *node density* on performance of different algorithms. All other parameters remain the same as in Fig. 1(a), except for $\mathrm{P_{max}}$ which was set to 1.5 in Fig. 3(a) to ensure network connectivity (lower node density requires higher transmission energy to form a connected network). We observe a consistent performance similar to what was observed in Fig. 1(a).

4) Effect of Transmission Power: In order to see the effect of transmission power $\mathrm{P_{max}}$ on energy cost, we set $\rho_0 = 0.2$, and simulate different values of $\mathrm{P_{max}}$. Results are shown in Figs. 4(a) and 4(b) for $\mathrm{P_{max}} = 2$ and $\mathrm{P_{max}} = 3$, respectively. Although the energy cost changes with changing $\mathrm{P_{max}}$, the relative energy cost behavior across different algorithms does not change.

5) Effect of Path Throughput: We fix $\mathrm{P_{max}}$ at $\mathrm{P_{max}} = 2$ and run the simulations with different values for ρ_0. Results from the simulations are shown in Figs. 5(a) and 5(b) for $\rho_0 = 0.1$ and $\rho_0 = 0.4$, respectively. We observe that the results under varying path throughput ρ_0 remain consistent with the results presented in Fig. 1(a).

As can be seen, the results are consistent with Fig. 1(a). In particular, CASPO significantly outperforms the other algorithms.

6) Optimal Cooperative Path: Cooperation along the shortest non-cooperative path is a widely used strategy for cooperative routing (CASP is an example). However, as our optimal routing formulation in Section III shows, the optimal cooperative route is not necessarily aligned with the non-cooperative route. The proposed 1-OPT algorithm provides a baseline to compare optimal and non-optimal cooperative routes, where the receiving set is limited to a single node (to avoid prohibitive simulation time). Fig. 6 shows

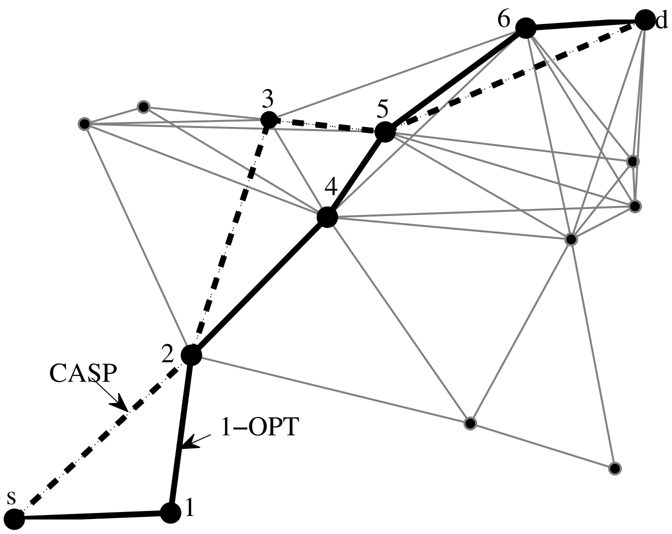
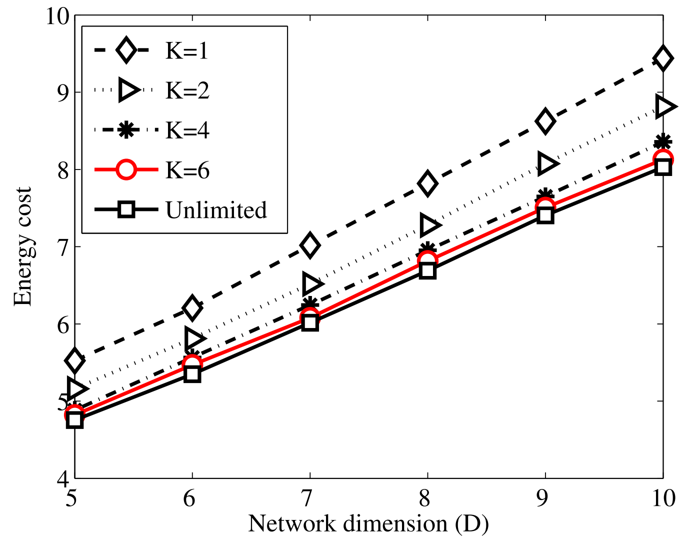


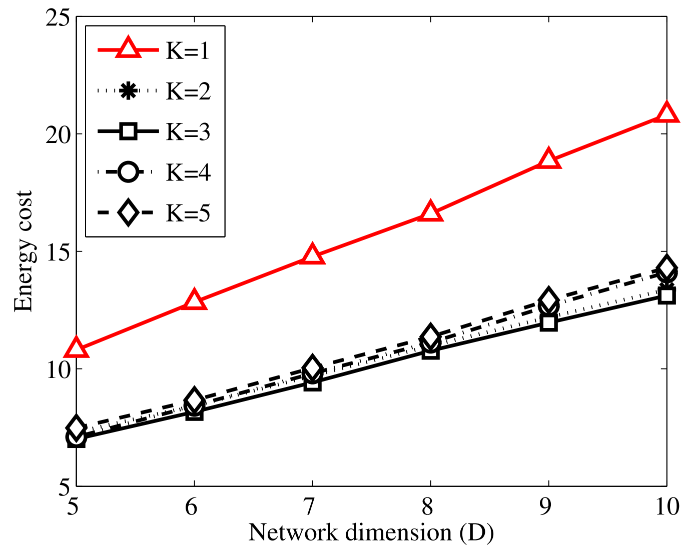
Fig. 6. Optimal versus heuristic cooperative routes.

a small network topology along with the cooperative routes $(s, 2, 3, 5, d)$ and $(s, 1, 2, 4, 5, 6, d)$ computed by CASP and 1-OPT respectively. In this example, 1-OPT achieves about 12% energy savings compared to CASP.

7) Limited Cooperation: Fig. 7 shows the performance of limited cooperative algorithms KT-CASPO and KR-CASPO for different values of K. It is observed from Fig. 7(a) that 6T-CASPO (*i.e.*, limiting the transmitting set to $K = 6$ nodes) achieves almost the same performance as CASPO, which uses unlimited transmitting sets. Similarly, Fig. 7(b) shows how energy cost changes as different receiving set sizes are used. In particular, only $K = 3$ receivers are sufficient to harness most of the gain of receiver diversity in KR-CASPO algorithm. These results can be used to find the appropriate size of transmitting and receiving sets in order to design efficient heuristic routing algorithms, as discussed earlier.



(a) Limited transmitters.



(b) Limited receivers.

Fig. 7. Limited cooperation algorithms.

V. CONCLUSION

In this paper, we explored cooperative diversity at the physical layer in order to develop energy efficient cooperative routing algorithms for wireless networks. Our network and routing models are appreciably general in that they subsume models considered by other researchers (*e.g.*, [5], [6]) such as single-input-single-output, single-input-multiple-output, and multiple-input-single-output models. We formulated the optimal routing problem and developed several heuristic routing algorithms that find energy efficient cooperative routes in polynomial time. Using simulations, we showed that the proposed algorithms are able to find energy efficient routes, and achieve significant energy savings compared to existing routing algorithms.

We emphasize that while this work provides useful tools and insights for designing energy-efficient cooperative routing algorithms, several issues remain to be addressed toward having a comprehensive cooperative routing algorithm:

- **Distributed Implementation and Protocol Design** By limiting the transmitting and receiving sets to neighboring nodes, a distributed routing algorithm can be designed. However, any implementation of the algorithm requires a protocol to form the transmitting and receiving clusters and determine the power allocation in a distributed manner. In particular, we did not discuss in this paper how to decide the power allocation in a distributed manner once the transmitting and receiving sets are chosen. A simple heuristic power allocation algorithm is to allocate power *equally* in the direction of each receiver. When transmitters and receivers are limited to neighboring nodes, this heuristic might provide a good approximation as channel gains are approximately similar in this case.

- **Multi-Flow Networks and Scheduling**
  In this paper, we only considered single flow networks, and developed models for power allocation and cooperative routing. However, we neither discussed joint routing and cooperation across different flows, nor did we discuss MAC-layer scheduling under the cooperative model. Specifically, due to cooperation, the nature of the interference is different from the interference caused by single node transmissions, and hence the scheduling problem requires special treatment.
- **Capacity Scaling**
  Capacity scaling of wireless networks has been subject to extensive research in the past few years (for example, see [13]–[15]). The latest results [16] indicate that by applying complicated communication schemes at the physical layer, such as the hierarchical cooperation discussed in [14], the capacity of a wireless network scales almost linearly with the number of nodes in the network. These results however are obtained for networks with many nodes. It would be interesting to understand how cooperation impacts the capacity of the networks that have small number of nodes.

APPENDIX

In the cooperative transmitting set, it is possible to have two transmitters, t_i and t_k, that are in the same distance from the receiving node r_j, *i.e.*, $d_{ij} = d_{kj}$. If these nodes are also allocated the same transmission power then the mean received power at receiver r_j from these transmitting nodes will be equal (see (5)). In this case, Π_{ij} can not be evaluated from (8) due to a division by zero and hence the outage probability can not be calculated.

To address this problem, we note that γ_j is the summation of m independent and exponentially distributed random variables with parameters $\lambda_{ij} = \frac{1}{\bar{\gamma}_{ij}}$. Therefore, γ_j follows a hypoexponential distribution:

$$\gamma_j = \sum_{i=1}^{m} \gamma_{ij} \sim \text{Hypoexponential}(\lambda_{1j}, \lambda_{2j}, \ldots, \lambda_{mj})\,.$$

Consequently, the CDF of γ_j can be expressed as

$$F_{\gamma_j}(y) = 1 - \alpha e^{y\mathbf{\Theta}}\mathbf{1},$$

where,

$$\mathbf{\Theta} = \begin{bmatrix} -\lambda_{1j} & \lambda_{1j} & 0 & \ldots & 0 & 0 \\ 0 & -\lambda_{2j} & \lambda_{2j} & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & -\lambda_{(m-1)j} & \lambda_{(m-1)j} \\ 0 & 0 & \ldots & 0 & 0 & -\lambda_{mj} \end{bmatrix},$$

and,

$$\alpha = (1, 0, \ldots, 0)\,.$$

Also, $\mathbf{1}$ is a unit column vector of size m, and $e^{\mathbf{A}}$ denotes the matrix exponential of $\mathbf{A}$.

Next, following (10), the outage probability for power allocation $\mathbf{p}$ and transmission rate λ is obtained as follows

$$\begin{aligned} \wp_j(\mathbf{p}, \lambda) &= \mathbb{P}\left\{\gamma_j < 2^{\lambda} - 1\right\} \\ &= F_{\gamma_j}(2^{\lambda} - 1) \\ &= 1 - \alpha e^{(2^{\lambda}-1)\mathbf{\Theta}}\mathbf{1}\,. \end{aligned}$$

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