On the Optimality of Opportunistic Routing Protocols for Underwater Sensor Networks

Full paper

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ABSTRACT

In the last decade, underwater wireless sensor networks (UWSNs) have attracted a lot of attention from the research community thanks to their wide range of applications that include seabed mining, military and environmental monitoring. With respect to terrestrial networks, UWSNs pose new research challenges such as the three-dimensional node deployment and the use of acoustic signals. Despite the large number of routing protocols that have been developed for UWSNs, there are very few analytical results that study their optimal configurations given the system's parameters (density of the nodes, frequency of transmission, etc.). In this paper, we make one of the first steps to cover this gap. We study an abstraction of an opportunistic routing protocol and derive its optimal working conditions based on the network characteristics. Specifically we prove that using a *depth threshold*, i.e., the minimum length of one transmission hop to the surface, is crucial for the optimality of opportunistic protocols and we give a numerical method to compute it. Moreover, we show that there is a critical depth threshold above which no packet can be transmitted successfully to the surface sinks, which further highlights the importance of properly configuring the routing protocol. We discuss the implications of our results and validate them by means of stochastic simulations on NS3.

CCS CONCEPTS

• **Networks → Network protocol design**; **Network performance modeling**; **Routing protocols**;

KEYWORDS

Underwater Wireless Sensor Networks, Modelling of Opportunistic Routing Protocols, Performance Evaluation

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1 INTRODUCTION

In the last decade, underwater wireless sensor networks (UWSNs) have attracted a lot of attention from the research community thanks to their important application fields which include environmental monitoring, military surveillance, and seabed exploration. UWSNs still pose several research challenges due to their peculiar characteristics: i) the adoption of acoustic transmission, ii) the high mobility of the nodes, iii) the low data rate available and the low speed of propagation, iv) the limited battery capacity of the motes and the difficulty in replacing or recharging them. Beside understanding the characteristics of the physical layer that are quite different from those of traditional radio-frequency based terrestrial networks (see e.g., [5, 10, 19]), designing efficient and reliable routing protocols is crucial for the development of these networks. To some extent, the routing protocols devised for UWSNs inherit some of the characteristics of those developed for vehicular networks, especially for those in which nodes are unaware of the position. In the UWSNs that we consider, the goal of routing is finding a multi-hop route from any underwater mote to the sonobuoys that float on the surface (see Figure 1). While flooding-based routing protocols such as the well-known AODV cannot be adopted due to the high variability of the nodes's locations, in UWSNs we have the advantage that motes can easily estimate their depth thanks to onboard pressure sensors. Therefore, most of the protocols aim at devising a controlled flooding strategy (see, e.g., [3] and the references therein) with the objectives of covering the longest distance with one hop and reducing the total energy consumption of the network.

Opportunistic routing protocols for UWSNs are classified as *receiver-based* and *sender-based* [6]. For receiver-based algorithms, a node that receives a packet decides if it is going to be a forwarder based on some factors among which the depth difference between itself and sender usually plays the major role (other factors may be

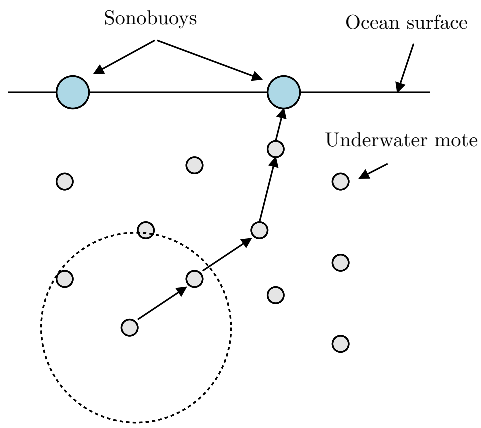

Figure 1: Sketch of a UWSN.

the residual energy in a battery). Sender-based algorithms, on the other hand, are more difficult to implement and require the sender node to specify which neighbour node(s) will be the forwarder(s). In this paper we focus on receiver-based routing algorithms and assume that the only factor that is used to decide the forwarding is the depth difference between the sender and the receiver. This is the case for many protocols including the *depth based routing* (DBR) that is still one of the mostly used protocols in actual implementations of UWSNs and is is adoped as benchmark for the performance evaluation of new routing protocols (see, e.g., [16, 20]).

Suppose that a node with depth difference x from the sender correctly receives a packet, and let $p(x)$ be the probability that it forwards it in the direction of the surface. The first problem that we address is the derivation of the optimal $p(x)$ conditioned on the expected number of forwarders v. Having more than 1 expected forwarder may be useful to create redundant routes to the surface and to increase the packet delivery ratio. Our first finding is that the optimal form of $p(x)$ is threshold based, i.e., $p(x) = 1$ if the depth difference is above a certain threshold and $p(x) = 0$ otherwise. This result holds under very mild assumptions on the topology of the network, since we require only the stationarity of the point process modelling the nodes' positions. A second contribution consists in giving an analytical formula and a numerical algorithm for the computation of the optimal threshold when the nodes form a homogeneous Poisson point process (PPP). Interestingly, protocols like DBR already introduced the idea of a depth threshold under which nodes are not eligible forwarders but, to the best of our knowledge, this is the first time that the optimality of that choice is proved in the three-dimensional environment of UWSNs and that an efficient numerical approach for the computation of the optimal threshold is given. In fact, the derivation of the optimal threshold based on stochastic simulations may be very time consuming.

The third contribution that we give is the estimation of the packet delivery probability given the expected number of forwarders for large UWSNs assuming a stationary PPP distribution of the nodes. We base our analysis on the theory of percolation on trees and derive a simple formula that, combined with the above mentioned results, allows one to fix a target packet delivery ratio and obtain the optimal threshold that allows that quality of service.

Finally, we address a case study of a realistic UWSN and compare the analytical results with the simulations estimates obtained with Aquasim-NG simulator [14], a Network Simulator (NS3) library for the analysis of underwater networks. The simulator was designed and verified by one of our previous research works [11] and provides a detailed simulation model of underwater sensors. The tool is open source and can be downloaded freely [13]. Interestingly, although the model does not take into account the mobility of nodes, the comparison with the simulations shows that it is robust with respect to node mobility. We discuss the motivations in Section 4.

The paper is structured as follows. Section 2 proves the optimality of the threshold based form of the forwarding probability $p(x)$ and gives the algorithms for the computation of this threshold in the case of homogeneous Poisson processes. In Section 3, we relate the expected number of forwarders with the packet delivery probability by resorting to percolation theory. In Section 4, we assess the accuracy of our analytical model with respect to stochastic simulations performed in NS3. Finally, in Section 6, we give some final remarks.

2 THRESHOLD BASED OPTIMISATION

In this section we study an UWSN employing a threshold based opportunistic protocol (see, e.g., [16, 20, 21]). We study an abstract formulation of such protocols that works in this way: each node that correctly receives a packet decides if it forwards the packet based on the depth difference between the sender and the receiver. As mentioned before, in UWSNs, nodes can easily estimate their depth by means of pressure sensors but can hardly know their absolute location. The depth difference x between the sender and the receiver is computed thanks to the depth information that the sender inserts in the packet. Let $p(x)$ be the probability that a node with depth difference x from the sender will forward the packet, and let v be the desired expected number of forwarders.

The main result that we prove in this section is that in order to maximise the distance to the surface covered by a transmission, $p(x)$ is always a step-function, i.e., $p(x)$ is 0 below a depth threshold T and is 1 otherwise. *This result is independent of the characteristics of the stochastic process modelling the locations of the nodes and of the function modelling the probability of a successful transmission.* We also provide a numerical method to compute the optimum threshold given the density of the nodes and the probability of error in the transmissions.

2.1 Modelling assumptions and goals

We consider an underwater sensor network deployed in a three-dimensional space. The spatial process modelling the sensor deployment is arbitrary but motion-invariant. Each node is aware of its depth thanks to a pressure sensor embedded in the node and we assume the surface to be covered by sink nodes that collect the packets harvested underwater (e.g., sonobuoy). We aim at designing a stateless multi-hop routing protocol capable of connecting each sensor node with the surface.

The stateless multi hop protocol works as follows:

- A node n broadcasts a message with the harvested data and specifies its own depth in the packet
- All the nodes that potentially receive the message and whose depth is lower than that of the transmitter are candidate forwarders. Let us call this set of nodes $\mathcal{S}(n) = \{n_1, n_2, \ldots\}$
- Each node correctly receives the packet with a probability $f(d)$ that depends on the Euclidean distance between the sender and the receiver $f(d) : \mathbb{R}^+ \to [0, 1]$, with $f(d)$ strictly monotonically decreasing and
$$\lim_{d\to\infty} f(d) = 0\,;$$
- Each node $n_i \in \mathcal{S}$ that has correctly received the packet computes the depth difference Δ_i between itself and the sender and retransmits (i.e., forwards) the packet with a probability that depends only on this value. Let us call this probability $p(x) : \mathbb{R}^+ \to [0, 1]$, where x is the depth difference

We aim at determining $p(x)$ such that, given the expected number of forwarders v, the expected distance covered in one transmission is maximised.

We assume that the nodes are distributed in $\mathbb{R}^3$ according to a point process (PP) Φ that satisfies some conditions that we will discuss hereafter. Φ is a countable random collection of points in $\mathbb{R}^3$. Let $\mathcal{B}^3$ be the Borel sets, then the σ-algebra consists of $\mathcal{B}^3$ with the Lebesgue measure. For each $B \in \mathcal{B}^3$, let $N(B)$ be the counting measure, i.e., a random variable associated with the number of nodes present in B. We define $\Lambda(B) = E\{N(B)\}$ as the intensity measure of Φ, and we assume it to be diffuse and to admit intensity function $\lambda(b)$ such that for each $B \in \mathcal{B}^3$:

$$\Lambda(B) = \int_B \lambda(b)db\,.$$

Φ is locally finite, i.e., for all B such that $|B| < \infty$ we have that $N(B) < \infty$, where $|B|$ is the Lebesgue measure of B. We assume Φ to be simple, i.e., $N(\{x\}) \in \{0, 1\}$ almost surely for all $x \in \mathbb{R}^3$, and stationary. As a consequence, $\Lambda(B) = \lambda|B|$ for some intensity $\lambda \in \mathbb{R}^+$.

Let $R \in \mathbb{R}^+$ be the transmission radius of a node n, i.e., we assume that for Euclidean distances larger than R the effect of the signal transmitted by n is negligible.

2.2 The model for general stationary point processes

Let us assume that the sender node is at location o. Let Φ_o be the PP obtained by conditioning on the presence of a point in o and that contains only the nodes that correctly receive the transmission from o. Formally, we are computing the reduced Palm distribution of Φ conditioned on o and then a thinning. Thus, Φ_o has diffuse density measure inside the sphere with radius R and centre o.

Since the nodes take their decisions of being forwarders based on the depth difference with the sender, we project Φ_o restricted to half-sphere with centre o and radius R on the vertical direction toward the surface, i.e., we consider the mapping function $\zeta : \mathbb{R}^3 \to [-R, R]$ and consider just the points in $[0, R]$:

$$\zeta(y) = y|_1 - o|_1 \tag{1}$$

where $y \in \mathbb{R}^3$, and $y|_1$, $o|_1$ denote the depth component of y and o, respectively. Now, let $\lambda_o(x)$ be the intensity function of this process. Then, the expected number of forwarders v is given by:

$$v = \int_0^R \lambda_o(x)p(x)dx\,, \tag{2}$$

and the expected distance covered by one transmission is:

$$\ell = \int_0^R x\lambda_o(x)p(x)dx\,. \tag{3}$$

Given the expressions for ℓ and v, we want to determine the optimal function $p(x)$ which selects the relay nodes based on the depth difference that gives the desired expected number of forwarders and maximises the distance covered by a single hop transmission. The problem is solved in Theorem 2.1.

THEOREM 2.1. *Given the optimisation problem:*

$$\text{maximise:} \quad \int_0^R x\lambda_o(x)p(x)dx \tag{4}$$

$$\text{subject to:} \quad 0 \le p(x) \le 1\,, \tag{5}$$

$$\int_0^R \lambda_o(x)p(x)dx = v \tag{6}$$

where $\lambda_o(x)$ is strictly positive everywhere in $[0, R)$. Then, problem (4) has a unique solution $p^*(x)$ defined as follows:

$$p^*(x) = \begin{cases} 0 & \text{if } x < T \\ 1 & \text{if } x \ge T \end{cases}\,, \tag{7}$$

whenever there exists a T in $[0, R]$ such that:

$$\int_T^R \lambda_o(x)dx = v\,. \tag{8}$$

Proof. Let

$$\Lambda_o(x) = \int_0^x \lambda_o(u)du\,,$$

and observe that $\Lambda_o(x)$ is strictly monotoniccally increasing thanks to the assumptions on $\lambda_o(x)$ and let us define $t = \Lambda_o(x)$ which implies $dt = \lambda_o(x)dx$. Notice that $\Lambda_o(x)$ is monotoniccally increasing and hence invertible. Therefore, Equation (4) can be rewritten as:

$$\int_0^{\Lambda_o(R)} \Lambda_o^{-1}(t)p(\Lambda_o^{-1}(t))dt$$

and the constraints become:

$$0 \le p(\Lambda_o^{-1}(t)) \le 1$$

$$\int_0^{\Lambda_o(R)} p(\Lambda_o^{-1}(t))dt = v\,.$$

Given $Z(t) = p(\Lambda_o^{-1}(t))$ we can rewrite the optimisation problem as:

$$\text{maximize:} \quad \int_0^{\Lambda_o^{-1}(R)} \Lambda_o^{-1}(t)Z(t)dt \tag{9}$$

$$\text{subject to:} \quad 0 \le Z(t) \le 1 \tag{10}$$

$$\int_0^{\Lambda_o(R)} Z(t)dt = v \tag{11}$$

Now, since $\Lambda_o^{-1}(x)$ is also monotonically increasing as $\Lambda_o(x)$, the unique solution to the optimisation problem is:

$$Z(x) = \begin{cases} 0 & \text{if } x < T' \\ 1 & \text{if } x \ge T' \end{cases}$$

whenever there exists T' such that:

$$\int_{T'}^{\Lambda_o(R)} Z(t)dt = v\,.$$

Now, recall that $Z(x) = p(\Lambda_o^{-1}(x))$ and that $\Lambda_o^{-1}(x)$ is strictly monotonically increasing, therefore the optimum $p^*(x)$ must follow definition (7), where $T = \Lambda_o^{-1}(T')$. □

2.3 The model for homogeneous Poisson point processes

In the previous section we proved Theorem 2.1 for a general stationary PP. Now, we assume that Φ is a homogeneous Poisson point process (PPP) and provide a method for the computation of the optimal threshold T. Recall that a homogeneous PPP is motion-invariant, therefore Theorem 2.1 is still valid. In general, an analytical expression for T can be derived only for some instances of $f(x)$, whereas a numerical approach must be adopted in the other cases.

Let Φ be a homogeneous Poisson point process (PPP) with intensity λ, i.e., the number of nodes in each set $B \in \mathcal{B}^3$ has distribution:

$$Pr\{N(B) = k\} = \frac{(\lambda|B|)^k}{k!} e^{-\lambda|B|}\,,$$

for $k \geq 0$, and for $B_1, B_2 \in \mathcal{B}$, such that $B_1 \cap B_2 = \emptyset$ we have that $N(B_1)$ is independent of $N(B_2)$. If we condition the process on the presence of a transmitting node at o, then, by Slivnyak's theorem [9, Thm. 8.10] the resulting Palm distribution does not change with respect to that of a PPP with the same intensity where we add a node in o. Without loss of generality, let us assume that o is at the origin of the axes. Following the lines of Section 2.2, we first obtain process Φ'_o with the process of thinning, i.e., we select the points that correctly receive the packet. A node at y receives the packet sent from the o with probability $f(||y||_2)$, where $||y||_2$ denotes the L2-norm of y. Notice that, we assumed that for $||y||_2 > R$, we have $f(||y||_2) = 0$. Therefore Φ'_o is a non-homogeneous point process in the sphere with centre o and radius R. More specifically, by the thinning theorem for PPPs [9, Thm. 2.36], since the thinning function depends only on the location of a point, then Φ' is a PPP with intensity function:

$$\lambda'_o(y) = \lambda f(||y||_2)\,.$$

Finally, we define another PP by mapping Φ'_o restricted to the half sphere that contains the points closer to the surface than o (i.e., those whose vertical components are positive). This mapping produces a non-homogeneous PPP in $[0, R]$ by the mapping theorem [9, Thm. 2.34] with intensity function:

$$\begin{aligned}\lambda_o(x) &= \int_0^{\sqrt{R^2-x^2}} 2\int_{-u}^{u} \sqrt{\frac{u^2}{u^2-z^2}}\lambda'\left(x, z, \sqrt{u^2-z^2}\right) dz\, du\\ &= \int_0^{\sqrt{R^2-x^2}} 2\int_{-u}^{u} \sqrt{\frac{u^2}{u^2-z^2}}\lambda f\left(\sqrt{x^2+u^2}\right) dz\, du\\ &= \int_0^{\sqrt{R^2-x^2}} 2\pi\lambda u f\left(\sqrt{x^2+u^2}\right) du\,.\end{aligned}$$

We can simplify the expression of the integral by a change of variable $\alpha^2 = x^2 + u^2$, i.e., $\alpha d\alpha = udu$, thus obtaining:

$$\lambda_o(x) = 2\pi\lambda \int_x^R \alpha f(\alpha)d\alpha\,. \tag{12}$$

Henceforth, we call

$$b(x) = \int_x^R \alpha f(\alpha)d\alpha\,,$$

and hence we can rewrite $\lambda_o(x) = 2\pi\lambda b(x)$.

Remark 1. *Can we assume $R \to \infty$? Notice that Equation (12) depends on R only through $b(x)$. Indeed, the integral $b(x)$ becomes improper for $R \to \infty$, and for Equation (12) to be meaningful we must have that $\lim_{R\to\infty} b(x) < \infty$. A sufficient condition for this to happen is that*

$$\lim_{\alpha\to\infty} \frac{\alpha f(\alpha)}{\alpha^p} = \frac{f(\alpha)}{\alpha^{p-1}} < \infty$$

for $p > 1$, which is usually true in real world applications.

2.4 Determining the optimal threshold for homogeneous PPPs

In order to determine the optimal threshold T, we need to solve Equation (8) in T. In the case of PPP we can rewrite Equation (8) as:

$$\int_T^R b(x)dx = \frac{v}{2\pi\lambda}\,,$$

let $v_p = v/(2\pi\lambda)$. We can write $b(x)$ as:

$$b(x) = \int_0^R \alpha f(\alpha)d\alpha - \int_0^x \alpha f(\alpha)d\alpha\,.$$

If $f(\alpha)$ is continuous in $[0, R]$ we define $H(x)$ such that $H''(x) = xf(x)$. Notice that:

$$\int_0^y \int_0^x H''(\alpha)d\alpha dx = H(y) - H(0) - yH'(0)\,.$$

Thus, we can rewrite Equation (8) as:

$$\begin{aligned}v_p = R\int_0^R \alpha f(\alpha)d\alpha &- \int_0^R \int_0^x \alpha f(\alpha)d\alpha\, dx\\ &- T\int_0^R \alpha f(\alpha)d\alpha + \int_0^T \int_0^x \alpha f(\alpha)d\alpha\, dx\,,\end{aligned}$$

i.e., by using function $H(x)$:

$$\begin{aligned}v_p = R(H'(R) - H'(0)) &- (H(R) - H(0) - RH'(0))\\ &- T(H'(R) - H'(0)) + H(T) - H(0) - TH'(0)\,,\end{aligned}$$

which reduces to:

$$H(T) - TH'(R) = v_p + H(R) - RH'(R)\,. \tag{13}$$

Observe that we can write the residual of Equation (13) as:

$$r(T) = H(T) - TH'(R) - v_p - H(R) + RH'(R)\,,$$

and hence:

$$\frac{\partial r(T)}{\partial T} = H'(T) - H'(R)\,.$$

Since $H'(x)$ is strictly monotonically increasing and $0 \leq T \leq R$, we conclude that $r(t)$ is monotonically decreasing in $[0, R]$ and hence it is particularly simple to find its unique root if one exists.

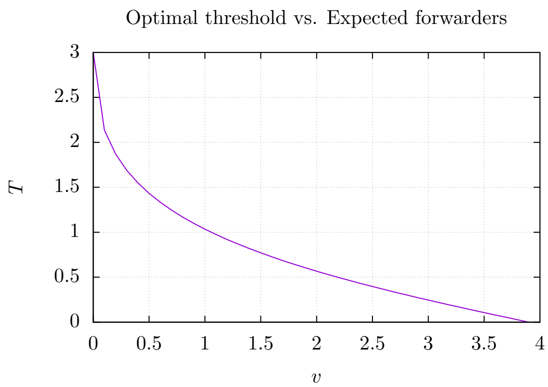

Figure 2: Optimal threshold as function of the expected number of forwarders for the model considered in Example 2.3.

Proposition 2.2. *If the optimisation problem considered in Theorem 2.1 admits a solution, then this is unique.*

Therefore, whenever $H(R), H'(R), H''(R)$ have symbolic expressions, one may derive the optimal threshold easily.

Example 2.3 (Computation of the optimal threshold T). We consider an example in which the probability of correct packet reception decays exponentially with the Euclidean distance from the source node, i.e., $f(x) = e^{-ax}$ for some parameter $a > 0$. Then, we have:

$$H''(x) = xe^{-ax}\,, \quad H'(x) = -\frac{e^{-ax}(ax+1)}{a^2}\,,$$
$$H(x) = \frac{e^{-ax}(ax+2)}{a^3}\,.$$

Therefore, in order to find the optimal threshold we nee to solve the following non-linear equation:

$$e^{-\alpha T}\frac{\alpha T+2}{\alpha^3} + e^{-\alpha R}\frac{\alpha T + 2 - \alpha^2 R^2 + \alpha^2 RT - 2\alpha R + \alpha T + 2}{\alpha^3} - v_p = 0\,.$$

For instance, let us assume a network with a density of 2 nodes per km^3 and let $\alpha = 1.8$, with $R = 3km$. Notice that at $3km$ of distance, the probability of correct reception is $4.5 \cdot 10^{-3}$. Then, the optimal depth threshold T for having 2 expected forwarders is $T = 0.567km$. It is interesting to note that if the intensity is 1 node per km^3 then, with the same parameters, it is impossible to achieve the goal of 2 expected forwarders. In Figure 2 we show the optimal threshold T as function of the expected number of forwarders for this example.

In many practical cases, the symbolic expression of $H(t)$ and $H'(t)$ is not known and hence the explicit or numerical solution of Equation (13) is not computationally feasible. Nevertheless, we can reconsider the original problem, i.e.:

$$r(T) = v_p - \int_T^R b(x)dx = 0\,,$$

and we recall that $r(T)$ is monotonic decreasing in $[0, R]$. Therefore, the solution for T may be easily numerically found thanks to the Newton-Raphson iteration scheme:

$$\begin{cases} T^{(0)} = \dfrac{R}{2}\,, \\ T^{(n+1)} = T^{(n)} - \dfrac{v_p - \int_{T^{(n)}}^R b(x)dx}{b(T^{(n)})}\,, \quad n \geq 0 \end{cases} \tag{14}$$

where at the denominator we need to numerically evaluate the integral:

$$\int_{T^{(n)}}^R \alpha f(\alpha)d\alpha\,,$$

while at the numerator:

$$\int_{T^{(n)}}^R \int_x^R \alpha f(\alpha)d\alpha dx\,,$$

that can be evaluated efficiently since the inner integrand function does not depend on x.

3 UWSN PERFORMANCE EVALUATION

In this section, we study the impact of the threshold on some performance indices of the network. The analysis relies on the following assumptions:

- **H1** The numbers of forwarders for each node are independent random variables;
- **H2** The network is sufficiently large to assume that it has an infinite number of nodes on a full three-dimensional space (i.e., nodes are not deployed on a plane);
- **H3** Nodes form a stationary PPP;
- **H4** The network protocol implements a perfect contention mechanism, i.e., the impact of collisions on the system's performance is negligible.

Hypothesis H1 tends to be verified thanks to the characteristics of UWSNs. In fact, the collision avoidance mechanism introduces some (possibly random) delays between the instant a node receives a packet and the moment in which this is forwarded (see e.g., [21]). As a consequence, if these delays are sufficiently large, the network topology may change between two different forwarding events. Finally, for dense networks, the number of nodes above the threshold that correctly receive the packet strongly depends on the probability of correct reception which is assumed to depend only on the distance. Hypotheses H2 and H3 are required to apply the percolation theory in order to estimate the packet delivery probability, and represent the typical scenario in this type of analysis. Finally, H4 is reasonable for delay tolerant UWSNs. There are several mechanisms that have been introduced to avoid collisions in redundant transmissions but before mentioning them we should recall that packets in UWSNs tend to be rather small, in the order of 300 − 400 bits and that the transmission time has a non negligible component which depends on the low speed of propagation of the acoustic signal. In order to avoid collisions in the packet forwarding, most of the protocols tend to avoid carrier sensing with the aim of reducing the power consumption, however a statistical or deterministic contention of the channel may be adopted. In the statistical contention, the nodes wait for a random time before forwarding the packet. In this case, the trade-off is between larger delays that reduce the probability of collisions and short delays that improve the system's response time. Another approach is based on the introduction of a deterministic delay whose duration is inversely proportional to the depth difference between the sending and receiving nodes. With respect to the statistical contention, this approach tends to reduce the transmissions' end-to-end delay by favouring the transmissions of the relay nodes that cover the longest distance in the direction of the surface.

3.1 Percolation analysis

Let us consider the stationary PPP modelling the nodes' locations in the UWSN and, without loss of generality, let the sender be located at the origin of the axis. Let V^T be the r.v. modelling the number of forwarders when the depth threshold is T, then the following proposition holds:

Proposition 3.1. *Given a depth threshold T, let $E[V^T] > 0$, then V^T is a Poisson random variable with intensity v.*

Proof. We already observed that the reduced Palm distribution obtained by conditioning on the location of the sender is still a stationary PPP with the same intensity of the original one by Slivnyak's theorem. We notice that, given T, the expected number of forwarders v is given by Equation (2), and by hypothesis $v > 0$. Since the probability that a node is a forwarder depends only on its location, by the thinning theorem for PPPs, the resulting process is a (possibly non-stationary) Poisson process. As a consequence, the number of nodes in a finite region B is a Poisson random variable. If we take B as the half-sphere with radius R whose points are closer to the surface than the sender node, we know that its expectation is v, and hence the distribution of the number of nodes is a Poisson r.v. with intensity v. □

Thanks to hypotheses H1 and H2, we use percolation theory to estimate the packet delivery probability. Assume that at time epoch 0 the node located at the origin sends a packet. This is forwarded by V^T nodes, where V^T is a Poisson r.v. with intensity v. Since the point process is stationary and thanks to H1 we can assume that the forwarding process in newly performed for each forwarder. Recall that, at each forwarding step, the relay nodes are closer to the surface than the sending node, i.e., the propagation of the packet is not monotone on the horizontal plane but is monotone in the direction of the surface. We estimate the packet delivery probability as the probability that the tree generated by the packet forwarding has infinite size.

Remark 2. *It is important to notice that the nodes forming the tree should not be intended as the nodes involved in the forwarding of the message, but rather as the transmissions involved. In fact, a node may be involved in the forwarding process multiple times. This is usually avoided in networks without node mobility, but it is a viable choice in UWSNs because of the high instability of the routes.*

Formally, let $Z_0 = 1$ be the initial transmission and Z_n the number of packet retransmissions that occur after n hops. Then, we have the following recursive relation:

$$Z_{n+1} = \sum_{i=1}^{Z_n} X_{n,i}\,, \qquad n \geq 0\,,$$

where $X_{n,i}$ is the random variable that models the number of forwarders of node i that transmitted after n hops. Clearly, by hypothesis $X_{n,i}$ are i.i.d. random variables whose distribution is the same as V^T, i.e., a Poisson r.v. with intensity v. Such a branching process is a Galton-Watson process [12]. The distribution of V^T is called offspring distribution.

We use the following two results:

(1) If $v \leq 1$ then the branching process does not grow forever with probability 1;[1]
(2) If $v > 1$ then the probability that the process grows forever is the minimum positive root of the equation $G(s) = s$, where $G(s)$ is the probability generating function of the offspring distribution. In our case, we need to find the minimum positive root of the following equation:

$$e^{v(s-1)} = s\,. \tag{15}$$

Example 3.2 (Packet delivery ratio and optimal threshold). Let us consider again the network with the parameters introduced in Example 2.3. We desire to study the optimal threshold as function of the desired packet delivery probability p_s. Given p_s, we may find the corresponding expected number of forwarders by solving Equation (15) and hence the optimal threshold T by solving Equation (13) or by applying the Newthon Rhapson algorithm (14). We show in Figure 3 the results for this example. It is important to note that when the expected number of forwarders $v \leq 1$ then the packet delivery probability is 0. Therefore we can identify a critical value for the threshold T that allows the connectivity of the UWSN with positive probability (see Definition 3.3). In our case, the critical threshold is $1.03474km$.

Definition 3.3 (Critical threshold). The critical threshold for a UWSN is the threshold whose corresponding expected number of forwarders is 1.

Clearly, for large UWSNs the depth threshold should never be higher than the critical threshold.

4 NUMERICAL RESULTS AND SIMULATION

In this section, we consider a real world scenario and evaluate the accuracy of the proposed method with respect to the estimates obtained by means of stochastic simulations. We start with the description of the simulation model, and then show the results obtained.

Packet delivery probability estimation. One pivotal element of our model is function $f(x)$, i.e., the probability that a packet is correctly received by a node placed at a distance x from the sender. In this paper, we use one of the most common models for the acoustic channel, i.e., the Stojanovic's model [5, 19]. According to this model

[1] Notice that in our case the offspring distribution is Poisson and hence the deterministic case is excluded.

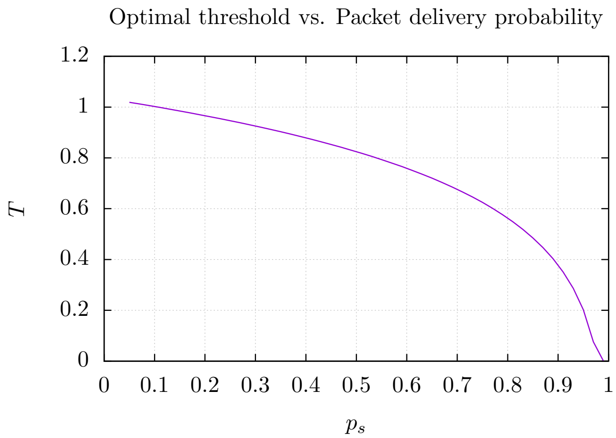

Figure 3: Optimal threshold as function of the packet delivery probability.

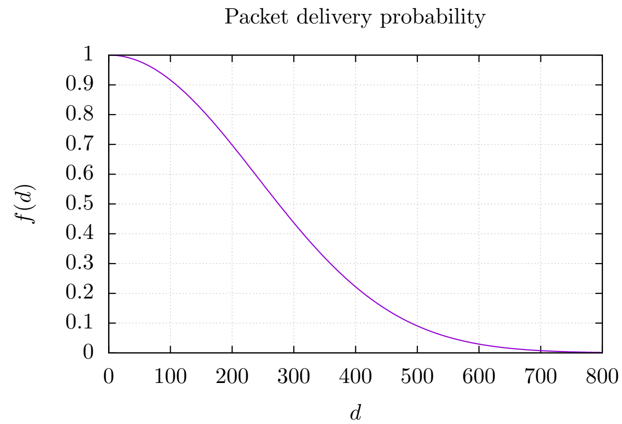

Figure 4: Packet delivery probability in the simulation model. d is measured in m.

the path loss over a distance d for a signal of frequency v is given by the following formula:

$$A(d, v) = d^k a(v)^d \,, \tag{16}$$

where $a(v)$ is the absorption coefficient and k is the spreading factor. For spherical spreading it is usually assumed that $k = 2$ (see, among others, [16]). $a(v)$ is obtained by using Thorp's formula [5]. In our setting, we used acoustic signals with $v = 10\,kHz$, so $a(v) = 1.1870\,db/km$. The average signal-to-noise ratio $\Gamma(d)$ at a distance d from the source node is given by:

$$\Gamma(d) = \frac{E_b / A(d, v)}{N_0} \,,$$

where $A(d, v)$ is computed by Formula (16), E_b is the average transmission energy per bit, and N_0 is the noise power density. We consider a small-scale Rayleigh fading effect modelled as in [19] where the probability of error for each bit is independent and computed as:

$$p_e(d) = \int_0^\infty p_e(x) p_d(x) dx \,, \tag{17}$$

where:

$$p_d(x) = \frac{1}{\Gamma(d)} e^{-x/\Gamma(d)} \,.$$

We assume the binary shift keying modulation which is widely used in modern acoustic modems. In [17] the expression for $p_e(d)$ is given as:

$$p_e(d) = \frac{1}{2}\left(1 - \sqrt{\frac{\Gamma(d)}{1 + \Gamma(d)}}\right) .$$

Since we assume packets of size $m = 320\,bits$ and independent errors, the probability that a node correctly receives a packet at distance d from the source is:

$$f(d) = (1 - p_e(d))^m \,. \tag{18}$$

Figure 4 shows the plot of $f(x)$ for the parameterization used in the simulations, when the transmission power is 140 db re μPa (see, e.g., [10]). As it is possible to see, we may take $R = 700m$.

Other details of the simulation model. In the network simulations, we use an UWSN with the routing protocol based on a threshold computed as discussed in Section 2. The simulation tool that we have used is specialised for underwater networks, namely AquaSim-NG [14], which is based on NS3 libraries [18]. The simulator accounts for the layered architecture of underwater network protocols and the operational modes of the modems. It is open source and freely downloadable from [13]. We consider a scenario with a network size of $2000x2000x2000$ meters, and the nodes are randomly deployed. We adopt the Gauss-Markov mobility model [2] for the nodes whose speed ranges from 1 to $3m/s$. Packets are always sent from the bottom of the network at a rate of 0.5 packets per second. We use the broadcast MAC protocol implementation proposed in [15]. In order to minimize the congestion, we use the holding time computed as in DBR [21]. Four on-surface sink nodes have been deployed at random positions. We assume a data rate of 100 kbps with a packet size of 40 bytes. Each estimate is the average of 15 independent experiments and in the plots we show the 98% confidence intervals.

Discussion of the results. Figure 5 shows the packet delivery probability $p_s(T)$ as function of the threshold T obtained with the analytical model for three different node densities. As expected, higher node densities allow for higher depth thresholds, although the relation is not linear as shown in Figure 6. The plot of Figure 5 clearly shows the critical thresholds for the three densities. Furthermore, it is interesting to observe that the reduction in the depth threshold for increasing the packet delivery probability from 0.7 to 0.8 is lower than that required for passing from 0.8 to 0.9. This suggests that the number of redundant paths required to achieve a high packet delivery probability may be quite high.

Figure 7 shows the validation of the model against the stochastic simulation for two node densities. We observe that the model predictions are rather accurate, although the values of the packet delivery probability tends to be over-estimated when the depth threshold is low. We can explain this with the observation that we are assuming a perfect contention mechanism for the channel whereas, when the number of expected forwarders increases, the

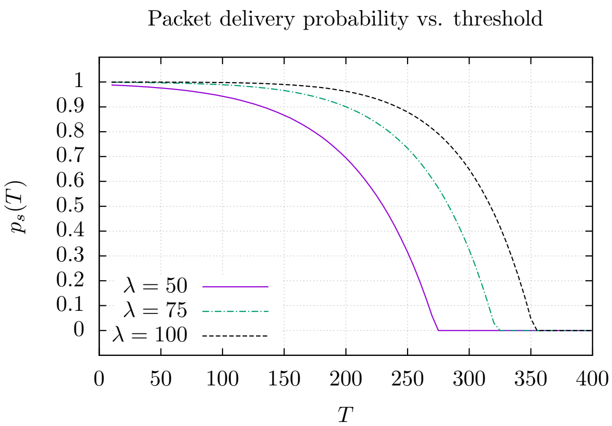

Figure 5: Analytical results: optimal threshold and packet delivery probability for different values of the node density expressed in $nodes/km^3$**,** T **is measured in** m**.**

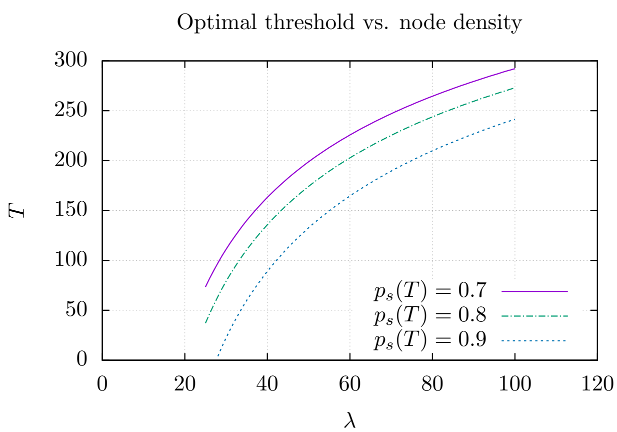

Figure 6: Analytical results: optimal threshold as function of the node density for different target packet delivery probabilities. T **is measured in** m **and** λ **in** $nodes/km^3$**.**

probability of hidden stations or interference with other delayed transmissions can worsen the system performance. Another important aspect to observe is that the simulation model takes into account the mobility of the underwater nodes. It may be surprising that the simulation estimates match closely the results of the analytical model that apparently does not contain any notion of mobility. We explain this effect by the *Displacement Theorem* for PPPs [Thm. 2.33][9] that basically states that in a PPP if the nodes are displaced by independent random vectors V_x at point x, then the resulting point process is still a PPP. Since we are using the stationary Gauss-Markov model [2], the conditions are satisfied and despite the mobility, the process of packet forwarding still occur in a different realization of the PPP but with the same statistics.

5 RELATED WORK

In the recent literature on underwater networks, opportunistic routing protocols have been a topic of prime importance as they are able

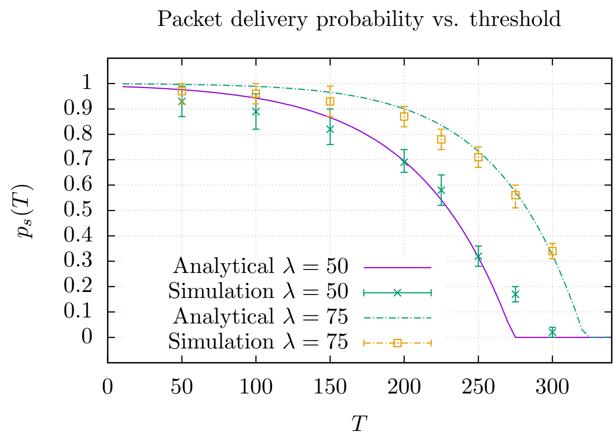

Figure 7: Validation of the model: analytical results and Aquasim-NG estimates for $\lambda = 50$ **and** $\lambda = 75$ $nodes/km^3$**.** T **is measured in** m**.**

to handle high node mobility. However, while they quickly react to the continuous changes in the routes towards the on-surface sinks, opportunistic protocols may be quite energy consuming. Therefore, the optimal configuration of opportunistic routing protocols given the network characteristics is widely recognised as a research topic of crucial importance. Coutinho et al. [6, 7] provide a detailed survey of opportunistic routing protocols, and classify them on the basis of their forwarding mechanisms. They also suggest some important design guidelines for the distributed routing schemes.

In [4] the authors propose a stochastic model to evaluate the total energy consumption and end-to-end delay of a UWSN under ideal conditions, however the model considers cylindrical propagation instead of the more common spherical that we use here. In Pigneri et al. [8] the authors focus their attention on the impact of interference on the network's performance. With respect to our work, the analytical model that they propose does not consider the multi-hop routing protocol implementation.

Zhou et al. [22] propose a multipath communication model for UWSNs in which they compute the number of possible paths for transmission between the source node and the sink. In this model, they identify the optimal power consumption for the single packet delivery, under the assumption that a node knows some network parameters such as path length and the number of available paths. This may be hard to achieve in real deployments due to the node mobility.

Apart from UWSNs, analytical models have also been devised in the area of vehicular networks to optimize the performance of opportunistic routing protocols. Probably the closest work to what we propose here is [1]. The authors provide a qualitative argument about the optimality of threshold based routing algorithm for terrestrial networks on a line. The assumptions use are stricter than those considered here, and the mono-dimensionality of the scenario does not allow one to trivially extend the result. In fact, in [1] the eligible forwarders have full information about the distance with the source node, while in our case they know only one of the three components (the vertical one, i.e., the depth difference) of the three-dimensional vector representing their relative positions.

Aside from these works, many other papers (see, e.g., [11, 16, 20, 21]) address the problem of the performance evaluation and optimisation of UWSNs protocol by means of stochastic simulation. Unfortunately, simulating large networks can be very time consuming and makes their optimisation very hard especially because the parameters depend on several characteristics of the networks such as the node density, the transmission frequency, the encoding used, just to mention a few.

6 CONCLUSION

Routing algorithms for UWSNs have a great impact on several performance indices including the end-to-end delay, the energy consumption and the packet delivery ratio. Most of the studies aimed at assessing the performance of these protocols resort to stochastic simulations which are usually very time consuming. In many cases, network optimisations based on these simulations are computationally prohibitive. In this paper, we propose an analytical model that allowed us to prove that the introduction of a depth threshold, similar to that proposed by DBR [21], is crucial for the definition of optimal routing protocols. The depth threshold T is used to control the number of forwarders by inhibiting the forwarding of the nodes whose depth difference with the sender is lower than T, and forcing it for all the nodes whose depth difference is above T. The analytical model allowed us to introduce an efficient method for the computation of the optimal depth threshold given a target packet delivery probability when the nodes' spatial distribution can be modelled by a homogeneous PPP. Despite the assumptions required by the model analysis, we showed that the NS3 Aquasim-NG simulations show estimates of the packet delivery probability that are quite close to those derived analytically. As future work, we plan to extend the model in order to include an estimation of the energy consumption associated with a certain threshold and hence to devise a numerical method to solve optimisation problems considering the trade-off between energy consumption and packet delivery ratio.

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