# Reliability Gain of Network Coding in Lossy Wireless Networks

Majid Ghaderi
Department of Computer Science
University of Calgary
mghaderi@cs.ucalgary.ca

Don Towsley and Jim Kurose
Department of Computer Science
University of Massachusetts Amherst
{towsley,kurose}@cs.umass.edu

***Abstract*—The capacity gain of network coding has been extensively studied in wired and wireless networks. Recently, it has been shown that network coding improves network reliability by reducing the number of packet retransmissions in lossy networks. However, the extent of the reliability benefit of network coding is not known. This paper quantifies the reliability gain of network coding for reliable multicasting in wireless networks, where network coding is most promising. We define the expected number of transmissions per packet as the performance metric for reliability and derive analytical expressions characterizing the performance of network coding. We also analyze the performance of reliability mechanisms based on rateless codes and automatic repeat request (ARQ), and compare them with network coding. We first study network coding performance in an access point model, where an access point broadcasts packets to a group of $K$ receivers over lossy wireless channels. We show that the expected number of transmissions using ARQ, compared to network coding, scales as $\Theta(\log K)$ as the number of receivers becomes large. We then use the access point model as a building block to study reliable multicast in a tree topology. In addition to scaling results, we derive expressions for the expected number of transmissions for finite multicast groups as well. Our results show that network coding significantly reduces the number of retransmissions in lossy networks compared to an ARQ scheme. However, rateless coding achieves asymptotic performance results similar to that of network coding.**

***Index Terms*—Reliability, network coding, ARQ, asymptotic analysis.**

## I. Introduction

In traditional networks, data packets are transmitted by store-and-forward mechanisms in which the intermediate nodes (relays or routers) only repeat data packets that they have received. With *network coding* (NC), a network node is allowed to combine several packets that it has generated or received into one or several outgoing packets. The original paper of Ahlswede *et al.* [1] showed the capacity gain of network coding for multicast in wireline networks. Recently, network coding has been applied to wireless networks and received significant attention as a means of improving network capacity and coping with unreliable wireless links [2]. In fact, the unreliability and broadcast nature of wireless links make wireless networks a natural setting for network coding.

In spite of significant research on the capacity gain of network coding, the reliability gain of network coding is largely unknown. In this paper, we study the application of network coding as an error control technique for reliable multicasting in a wireless network. Our goal is to quantify the benefit of using network coding compared to traditional error control techniques such as ARQ and rateless coding.

Recently, there has been some work on characterizing the reliability benefit of network coding in lossy networks [3]–[5]. However, our work differs from existing work in that we provide tight asymptotic bounds on the performance of reliability mechanisms based on both ARQ and NC. Additionally, the existing work only provides performance results in terms of the expected number of transmissions without providing any insight about the scaling behavior of different reliability mechanisms. Moreover, in addition to the commonly studied access point model, we analyze reliable multicast in a tree topology as well. Tree-based multicast has been previously studied in the context of wired networks and ARQ mechanisms [6], [7]. In this paper, we present analytical and numerical results for the performance of end-to-end and link-by-link reliability mechanisms based on ARQ, FEC and NC in a tree topology. An interesting model can be constructed by allowing the receivers of the traditional access point model to communicate locally in order to recover lost packets. This model provides an efficient structure for reliable multicast when the access point transmissions are costly or the communication quality among the receivers is superior to that of the access point, a scenario occurring often in military and satellite communications. Due to space limitations, analysis of the extended access point model is not presented in this paper. Interested readers are referred to [8] for details.

Our contributions in this paper are as follows:

- We present a detailed characterization of the performance of different reliability mechanisms based on ARQ, FEC and NC for the access point model and tree-based multicast model. We present both analytical and numerical results.
- We provide asymptotic bounds on the performance of different reliability mechanisms for the topologies considered in the paper, and show how our results can be used to analyze more complicated topologies.

The rest of the paper is organized as follows. In Section II, we analyze the access point model which serves as a basis for the analysis in following sections. In Section III, we study four different reliability mechanisms for multicast in a tree

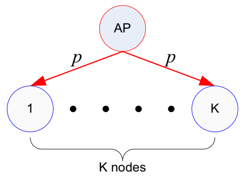


Fig. 1. The access point model with $K$ receivers.

topology. These mechanisms are based on the application of ARQ and FEC in a link-by-link or end-to-end fashion. For the three models considered here, we derive expressions for the expected number of transmissions, and provide asymptotic results for the performance of reliability mechanisms based on ARQ and NC. Our conclusions as well as future work are discussed in Section IV.

## II. Access Point Model

The access point model consists of a single source, called the access point (AP), broadcasting to a set of $K$ receivers over a lossy wireless channel as depicted in Fig. 1. Transmissions from the AP to receivers are lossy with losses distributed by independent identical Bernoulli processes with parameter $p$. We assume the use of block coding for NC, where $B$ denotes the size of the coding block. With NC, the AP transmits random linear combinations of the $B$ packets belonging to the same coding block. Hence, receivers need to receive $B$ independent linear combinations in order to decode the original packets (please see [9] for more information on random linear coding). Throughout this paper, we assume that feedback is reliable, and hence, do not consider the overhead and complexity of the feedback mechanism. Interested readers are referred to [8] for an analysis of the overhead of random linear network coding.

### A. Distribution of the Number of Transmissions

Let $N$ denote the number of transmissions of a packet by the AP until the packet is received by all $K$ receivers. It is straightforward to compute $\mathbb{P}\{N \le n\}$ as follows (for more details, see [6], [7] for ARQ, and [4], [5] for NC).

*1) ARQ Performance:* The probability that a node does not receive any packet out of $n$ packets transmitted by the AP is $p^n$. Therefore, with probability $1-p^n$ the node receives at least one of the packets. All $K$ nodes have independent losses, therefore, the probability that every node receives at least one packet is

$$\mathbb{P}\{N \le n\} = (1-p^n)^K, \quad n \ge 1\,. \tag{1}$$

*2) NC Performance:* We assume the block size for network coding is $B$. The probability that a node receives at least $B$ coded packets out of $n$ packets transmitted by the AP is given by a binomial distribution. Each node needs $B$ packets in order to decode and extract the original $B$ packets. Therefore, we obtain

$$\mathbb{P}\{N \le n\} = \left(\sum_{i=B}^{n} \binom{n}{i} (1-p)^i p^{n-i}\right)^K, \quad n \ge B\,. \tag{2}$$

### B. Asymptotic Analysis

The exact expressions derived in the previous subsection do not provide insight about the scaling behavior of the number of transmissions with respect to $K$ and $B$. In this subsection, we derive asymptotic expressions for the expected number of transmissions for the access point model. In particular, we are interested in the asymptotic performance of the ARQ and NC mechanisms as the number of receivers becomes very large, *i.e.*, $K \to \infty$. For NC, we only consider infinitely large block sizes and assume that $B = \Omega(K)$, namely block size grows to infinity faster than the number of receivers. Interested readers are referred to [10] for preliminary results on the performance of NC with finite block sizes, *i.e.*, $B = O(1)$.

Let $N_k$ denote the number of transmissions at the AP until node $k$ receives all the packets (one packet in ARQ, and $B$ packets in NC). We are interested in characterizing the following expectation as the performance metric

$$\mathbb{E}[N] = \mathbb{E}\left[\max_{1\le k\le K} N_k\right]. \tag{3}$$

*1) ARQ Performance:* In this case, $N_k$ has a geometric distribution. In other words, the probability that node $k$ receives the $n$-th transmitted packet is given by

$$\mathbb{P}\{N_k = n\} = (1-p)p^{n-1}\,. \tag{4}$$

Therefore, $\mathbb{E}[N]$ is the expected value of the maximum of $K$ geometric random variables. For the sake of analysis, we approximate each geometric random variable $N_k$ by an exponentially distributed random variable $X_k$ with rate $\mu$. In order to find $\mu$, we solve the equation $\mathbb{P}\{N_k \le n\} = \mathbb{P}\{X_k \le n\}$, which yields $\mu = -\ln p$. We then approximate $\mathbb{E}[N_k]$ by $\mathbb{E}[X_k]$. To compute the approximation error, denoted by $\epsilon$, we note that

$$\mathbb{E}[N_k] = \sum_{n=0}^{\infty} \mathbb{P}\{N_k > n\} \tag{5}$$

$$\mathbb{E}[X_k] = \sum_{n=0}^{\infty} \int_n^{n+1} \mathbb{P}\{X_k > x\}\, dx \tag{6}$$

Therefore,

$$\begin{aligned}\epsilon &= \mathbb{E}[N_k] - \mathbb{E}[X_k] \\ &= \sum_{n=0}^{\infty} p^n + \frac{1}{\mu}\sum_{n=0}^{\infty}(e^{-(n+1)\mu} - e^{-n\mu}) \\ &= \sum_{n=0}^{\infty} p^n + \frac{1}{\mu}\sum_{n=0}^{\infty}(p^{n+1} - p^n) \\ &= \frac{1}{1-p} + \frac{1}{\ln p}\,.\end{aligned} \tag{7}$$

Let $\{X_k\}$ $(k = 1, \ldots, K)$ denote a set of $K$ independent exponentially distributed random variables with parameter $\mu$. Then, using properties of exponentially distributed random variables, it follows that (see [10] for details)

$$\begin{aligned}\mathbb{E}[X] &= \mathbb{E}\left[\max_{1\le k\le K} X_k\right] \\ &= \sum_{k=1}^{K}\frac{1}{k\mu} = \frac{1}{\mu}H(K),\end{aligned} \tag{8}$$

where, $H(K)$ is the $K$-th *harmonic number*. It is well-known that for large $K$

$$\lim_{K\to\infty} H(K) = \ln K + \gamma, \tag{9}$$

where, $\gamma$ is Euler's constant. Hence, we have

$$\begin{aligned}\lim_{K\to\infty}\mathbb{E}[N] &= \lim_{K\to\infty}\mathbb{E}[X] + \epsilon \\ &= \frac{\ln K}{-\ln p} + \frac{\gamma}{-\ln p} + \frac{1}{1-p} + \frac{1}{\ln p}\,.\end{aligned} \tag{10}$$

From this we conclude that $\mathbb{E}[N] = \Theta(\log K)$, where $\log K = \frac{\ln K}{\ln 1/p}$.

*2) NC Performance:* In this case, $N_k$ has a negative binomial distribution, that is

$$\mathbb{P}\{N_k = n\} = \binom{n-1}{B-1}(1-p)^B p^{n-B}\,. \tag{11}$$

This means that $B-1$ packets have been received until packet $n-1$, and packet $n$ is received too. We are interested in characterizing the expected number of transmissions per packet:

$$\mathbb{E}[N] = \frac{1}{B}\mathbb{E}\left[\max_{1\le k\le K} N_k\right]\,. \tag{12}$$

However, it is difficult to compute $\mathbb{E}[N]$ using the negative binomial formulation in (11). Fortunately, we can represent $N_k$ by a different form, which is then amenable to analysis. In an alternative but equivalent form, $N_k$ can be considered as the sum of $B$ IID geometric random variables with parameter $(1-p)$. Each geometric random variable represents the number of transmissions at the AP until one of the $B$ packets is received at node $k$. That is

$$N_k = G_1^k + \cdots + G_B^k, \tag{13}$$

where, $G_n^k$ is the number of transmissions at the AP until node $k$ receives the $n$-th packet, given that it has already received $n-1$ packets. For geometric random variables we have

$$\mathbb{P}\left\{G_n^k = i\right\} = (1-p)p^{i-1}, \tag{14}$$

and, $\mathbb{E}\left[G_n^k\right] = \frac{1}{1-p}$. Now, we rewrite (12) as follows,

$$\begin{aligned}\mathbb{E}[N] &= \frac{1}{B}\mathbb{E}\left[\max_{1\le k\le K} N_k\right] = \mathbb{E}\left[\max_{1\le k\le K}\frac{N_k}{B}\right] \\ &= \mathbb{E}\left[\max_{1\le k\le K}\frac{G_1^k + \cdots + G_B^k}{B}\right]\,.\end{aligned} \tag{15}$$

We are interested in computing $\mathbb{E}[N]$ as $B\to\infty$. We have

$$\begin{aligned}\lim_{B\to\infty}\mathbb{E}[N] &= \lim_{B\to\infty}\mathbb{E}\left[\max_{1\le k\le K}\frac{G_1^k + \cdots + G_B^k}{B}\right] \\ &= \mathbb{E}\left[\max_{1\le k\le K}\lim_{B\to\infty}\frac{G_1^k + \cdots + G_B^k}{B}\right]\,.\end{aligned} \tag{16}$$

Note that the $G_n^k$'s are IID and hence, by applying the law of large numbers, we obtain

$$\lim_{B\to\infty}\frac{G_1^k + \cdots + G_B^k}{B} = \frac{1}{1-p} \tag{17}$$

By substituting into (16), we obtain

$$\begin{aligned}\lim_{B\to\infty}\mathbb{E}[N] &= \mathbb{E}\left[\max_{1\le k\le K}\frac{1}{1-p}\right] \\ &= \frac{1}{1-p} = \Theta(1)\,.\end{aligned} \tag{18}$$

### *C. Reliability Gain of NC*

Let $N_{\text{ARQ}}$ and $N_{\text{NC}}$ denote the number of transmissions in the case of ARQ and NC, respectively. Define the ***reliability gain*** of network coding as follows:

$$\text{Reliability Gain} = \frac{\mathbb{E}[N_{\text{ARQ}}]}{\mathbb{E}[N_{\text{NC}}]}, \tag{19}$$

where, $\mathbb{E}[N_{\text{ARQ}}]$ and $\mathbb{E}[N_{\text{NC}}]$ are given by (10) and (18), respectively. Then, the reliability gain of network coding for the access point model is of order $\Theta(\log K)$ as $K$ becomes large. In the following section, we show that the logarithmic gain of network coding is achievable in tree topology as well (in [8] we have shown that the same scaling holds for an extended access point topology in which receivers are allowed to communicate locally in order to recover lost packets without involving the access point).

## III. Multicast over a Tree Topology

In this section, we study the performance of different reliability mechanisms for reliable multicast over a tree topology as depicted in Fig. 2. We base our analysis on the access point model, and derive exact and asymptotic expressions for the reliability gain of network coding. The following reliability mechanisms are considered:

1) **End-to-End ARQ:** The root of the multicast tree retransmits each packet until it is correctly received by all the multicast receivers. All other nodes in the tree only forward packets they receive from their parents to their children.
2) **End-to-End FEC:** This technique is commonly referred to as rateless coding [11]. Similar to end-to-end ARQ, only the root of the multicast tree is responsible for retransmitting a packet until it is received by all receivers. All other nodes only forward the packets they receive from their parents to their children. For FEC-based schemes, we assume the use of a block coding technique to create coded packets for transmission.
3) **Link-by-Link ARQ:** Every node of the multicast tree is responsible for the reliable transmission of packets to its children. That is, a node retransmits the packet it has received from its parent to its children until the packet is correctly received by all of its children. Note that some children may receive more than one copy of the packet because of the random nature of packet losses.

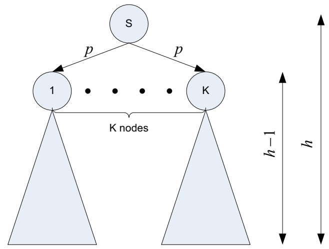


Fig. 2. Tree topology for reliable multicast.

4) **Link-by-Link FEC (NC):** We refer to this technique as network coding (NC) because coding is performed not only at the source but also within the network. That is, every node is responsible for reliable delivery of the block of packets it has received from its parent to its children. With NC, each node performs rateless coding to deliver a block of packets to its children.

Note that link-by-link reliability mechanisms are equivalent to the access point model that we studied in the previous section. Essentially, each node is responsible for the reliable delivery of the packets it has received from its parent to its children. Therefore, the expected number of transmissions at each node of the tree can be readily computed from (1) and (2), for ARQ and NC, respectively.

### A. Distribution of the Number of Transmissions

In this subsection, we study the performance of end-to-end reliability mechanisms based on ARQ and FEC. Let $N_r$ denote the number of transmissions of a packet to the root of a subtree of height $r$ (from its parent) before the packet is received by all nodes of the subtree. For the source of a multicast tree of height $h$, we interpret $N = N_h$ as the number of packet transmissions at the source until the packet is received by all of the multicast receivers. Define $F_r(n)$ as follows:

$$F_r(n) = \mathbb{P}\{N_r \le n\}, \qquad 0 \le r \le h, n \ge 1. \tag{20}$$

Similar to [6], we develop recursive relations to compute $F_r(n)$. First, consider the case of $r > 0$, and denote the root of the subtree by $s$. The probability that $i$ packets out of $n$ packets that have been transmitted to node $s$ are received by node $s$ is given by a binomial distribution,

$$\mathbb{P}\{i|n\} = \binom{n}{i}(1-p)^i p^{n-i}, \qquad 0 \le i \le n. \tag{21}$$

Note that for the root of the multicast tree, the error probability is zero, *i.e.*, $p = 0$, and hence $\mathbb{P}\{i|n\} = 1$, if $i = n$, and $\mathbb{P}\{i|n\} = 0$, otherwise. If node $s$ receives $i$ packets, it will broadcast the $i$ received packets to its children. For each child, the probability that all nodes of the subtree rooted at that child receive a packet is given by $F_{r-1}(i)$. Since the children of a node have independent packet losses, the probability that all of the nodes of the subtrees rooted at children of node $s$ receive a packet is given by $\{F_{r-1}(i)\}^K$, which we denote by $F_{r-1}^K(i)$ for notational simplicity. Therefore, by summing over all possible values of $i$, we obtain

$$F_r(n) = \sum_{i=0}^{n} \binom{n}{i}(1-p)^i p^{n-i} F_{r-1}^K(i), \qquad 0 < r < h. \tag{22}$$

Hence, we have a recursive equation for computing $F_r(n)$ for $r > 0$. Interestingly, computing $F_r(n)$ for $r > 0$ is independent of the applied reliability mechanism.

Next, we compute $F_0(n)$ for the leaves of the multicast tree as follows.

*1) End-to-End ARQ:* The probability that a (leaf) node does not receive any packet out of $n$ transmitted packets is given by $p^n$. Therefore, with probability $1 - p^n$ the node receives at least one copy of the packet. Therefore,

$$F_0(n) = 1 - p^n, \quad n \ge 1. \tag{23}$$

*2) End-to-End FEC:* The probability that a node receives at least $B$ coded packets out of $n$ transmitted packets is given by a binomial distribution. Therefore,

$$F_0(n) = \sum_{i=B}^{n} \binom{n}{i}(1-p)^i p^{n-i}, \qquad n \ge B. \tag{24}$$

So far, we have computed $F_r(n)$ for all subtrees of height $r$. As mentioned earlier, for the root of the multicast tree, we have $\mathbb{P}\{n|n\} = 1$. Hence, the expression for $F_h(n)$ can be simplified as $F_h(n) = F_{h-1}^K(n)$, where, $F_{h-1}(n)$ is given by (22). That is $\mathbb{P}\{N \le n\} = F_{h-1}^K(n)$.

### B. Expected Number of Transmissions

Using the expressions for $\mathbb{P}\{N \le n\}$, we can compute the expected number of transmissions at the root of the multicast tree, *i.e.*, $\mathbb{E}[N]$. Next, we compute the expected number of transmissions in the multicast tree (not just at the root) until a packet is received by all receivers. Let $T$ denote the total number of transmissions in the multicast tree until a packet is received by all receivers. We are interested in computing $\mathbb{E}[T]$.

*1) Link-by-Link Mechanisms:* Consider a multicast tree of height $h$. At height $r$ of the tree, there are $K^{h-r}$ nodes. For each of them, the expected number of transmissions ($\mathbb{E}[N]$) can be computed from (1) and (2), for ARQ and NC, respectively. This results in

$$\mathbb{E}[T] = \mathbb{E}[N] \sum_{r=1}^{h} K^{h-r} = \frac{K^h - 1}{K - 1}\mathbb{E}[N]. \tag{25}$$

Note that for $K = 1$, we have $\mathbb{E}[T] = h\mathbb{E}[N]$.

*2) End-to-End Mechanisms:* First, we compute the expected number of transmissions in the tree for each transmission at the root of the multicast tree. Let $X_r$ denote the number of transmissions in a subtree of height $r$ for each transmission at the root of the subtree. Then,

$$\begin{aligned} \mathbb{E}[X_r] &= 1 + \sum_{j=0}^{K} \binom{K}{j}(1-p)^j p^{K-j} \Big(j\mathbb{E}[X_{r-1}]\Big) \\ &= 1 + K(1-p)\mathbb{E}[X_{r-1}], \end{aligned} \tag{26}$$

where, $X_0 = 0$. The idea is that, for each packet received at the root of a subtree, there is one transmission at the root of the subtree. If $j$ children out of the $K$ children receive the packet, then in average, each of them will have $\mathbb{E}[X_{r-1}]$ transmissions in their subtrees. This yields

$$\mathbb{E}[X_r] = \frac{(Kq)^r - 1}{Kq - 1}, \tag{27}$$

where, $q = 1 - p$. Therefore, the expected number of transmissions per packet in the multicast tree is given by

$$\mathbb{E}[T] = \mathbb{E}[X_h]\,\mathbb{E}[N] = \frac{(Kq)^h - 1}{Kq - 1}\mathbb{E}[N]\ . \tag{28}$$

Note that if $Kq = 1$, then it follows that $\mathbb{E}[T] = h\mathbb{E}[N]$.

### C. Asymptotic Analysis

For the link-by-link reliability mechanisms, the same bounds we derived for the access point model apply to the tree toplogy as well. For end-to-end mechanisms, we consider the case of having $K$ exponentially growing to infinity, *i.e.*, $\log K \to \infty$. In this case, we can apply the results from the access point model to derive asymptotic expressions for the expected number of transmissions at the root of the multicast tree, and hence, within the tree.

*1) ARQ Performance:* Based on the analysis of the access point model, in order to deliver a copy of a message to all leaf nodes, *i.e.*, nodes at height 0, each node at height 1 needs to transmit the message $\Theta(\log K)$ times. The key idea is that because $\log K \to \infty$, the law of large numbers can be applied in a fashion similar to the access point model. Hence, the nodes at height 2 need to transmit only $c = \frac{1}{1-p}$ times more in order for each node at height 1 to receive $\log K$ copies of the packet. Therefore, we have

$$\mathbb{E}[N] = \Theta\Big(c^{h-1}\log K\Big)\,. \tag{29}$$

*2) FEC Performance:* We have the same argument for end-to-end FEC except that at each level of the tree, we need to retransmit a block of packets only a constant number of times, *i.e.*, $c = \frac{1}{1-p}$ times. Therefore, we obtain

$$\mathbb{E}[N] = \Theta\Big(c^{h}\Big)\,. \tag{30}$$

Table I has summarized the reliability gain of network coding compared to other reliability mechanisms for multicast in a tree topology. Interestingly, the asymptotic gain of network coding compared to end-to-end FEC, *i.e.*, rateless coding, is just a constant factor. However, depending on the height of the multicast tree ($h$), and the loss probability ($p$), the constant can be arbitrary large.

| Reliability Mechanism | Reliability Gain |
|---|---|
| Link-by-Link ARQ | $\Theta(\log K)$ |
| End-to-End ARQ | $\Theta(c^{h-1}\log K)$ |
| End-to-End FEC | $\Theta(c^h)$ |

TABLE I
RELIABILITY GAIN OF NC IN A TREE TOPOLOGY OF HEIGHT $h$ ($c = \frac{1}{1-p}$).

## IV. CONCLUSION

In this paper, we studied the benefit of network coding for reliable multicast in lossy wireless networks. We analyzed the access point model in which an access point broadcasts packets to a set of $K$ receivers. We showed that, for large coding blocks, the reliability gain of network coding compared to ARQ is of order $\Theta(\log K)$. We then used the access point model to study the reliability gain of network coding in a tree-based reliable multicast. For the tree topology, four different reliability mechanisms based on ARQ and NC were considered. We showed that NC achieves the best performance in terms of the required number of transmissions in the tree. We also extended the access point model by allowing inter receiver communication to recover lost packets, and showed that still the $\log K$ reliability gain can be achieved while minimizing the number of transmissions at the access point [8]. In the future, we would like to extend our analysis to more complicated network topologies such as a grid with significant amount of *path diversity*.

## V. ACKNOWLEDGEMENTS

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