Indoor Wireless Planning using Smart Antennas

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***Abstract*—This paper considers the problem of indoor wireless planning using *smart antennas*. Smart antennas have gained much attention in wireless networking because of their capability in providing more spatial reuse and increased network capacity. Recent research has demonstrated their effectiveness in *indoor* environments where omni-directional antennas have been traditionally the dominant technology. Much of the work, however, assumes that a network is already deployed and focuses on scheduling antenna patterns. In this work, we investigate finding a *wireless plan* for an indoor environment where the wireless plan specifies *minimum* number of antennas required to provide complete coverage of the environment as well as the location, transmission power and beam pattern for each antenna. This problem is more challenging than radio planning using omni-directional antennas because of the special shape of antenna beams. Both *single-beam* and *multi-beam* antenna patterns are considered and *Integer Linear Programming* formulations are provided for computing the minimum cost wireless plan. Moreover, to solve large-scale instances of the problem an efficient polynomial-time heuristic is proposed.**

I. Introduction

Emerging popularity of WiFi-enabled consumer devices in recent years, has resulted in growing demand for wireless Internet access in indoor environments such as university campuses and corporate buildings. The large number of users connected to such wireless LANs have to compete to gain access to the shared wireless medium. The high-rate of contention reduces the overall throughput of these networks. Moreover, laptops and other powerful handheld devices that are capable of running bandwidth-intensive applications constantly request higher data rates from the network. To address the limited bandwidth challenge, new technologies such as the *smart antennas* are being used to build more efficient wireless LANs [1] [3].

Usually a single access point (AP) [1] is not capable of providing satisfactory service for an indoor environment (*e.g.*, university campus), thus multiple APs are deployed to cover the area. In a multi-AP wireless LAN, transmitted signals might be degraded or even corrupted due to interference from transmissions of other APs. While according to *Signal to Interference-plus-Noise Ratio* (*SINR*) model [10], noise can have the same negative effect, interference is the dominant limiting factor for the performance of multiple transmitter wireless networks [2]. Thus, to enhance the performance of a wireless network, either the average received signal power should be increased or the interference power should be decreased. Smart antennas can serve both goals well.

[1]We use the terms *access point* and *antenna* interchangeably.

Smart antenna refers to an antenna array consisting of multiple omni-directional antenna elements, combined with smart signal processing algorithms [4]. Smart antennas have been extensively used in radar and aerospace systems, and most recently, in wireless communication systems. Perhaps the most important feature of a smart antenna is its *beamforming* capability. When beamforming to a user, the smart antenna creates a directional beam toward the desired user and null the signal in the directions of undesired users by appropriately adjusting the magnitude and phase of the signal transmitted by each of its elements. In comparison to omni-directional transmissions, beamforming reduces interference allowing more concurrent transmissions in the network. Moreover, by concentrating the transmission energy in a specific direction, beamforming creates a signal that is orders of magnitude stronger than that of the signals in other directions. One can further reduce interference in a network by combining beamforing feature with power control [5] and scheduling [1], [6].

While these techniques can significantly improve the performance of wireless networks, the amount of interference in a wireless network, and consequently its performance, fundamentally depends on the *network geometry* [2]. The process of finding the optimal network geometry, which naturally happens during the planning phase of a network, is what we call the *wireless planning problem* (WPP). In this work, we provide a mathematical optimization framework to systematically solve this problem. In our framework, the objective is to find the optimal network geometry determined by antennas' configuration in terms of antenna *location* and *beam pattern* as well as *transmission power* so that a minimum pre-specified SINR is observed across the network.

There exist several work on wireless planning mainly in the context of omni-directional antennas [7] [8]. Osais *et al.* [9] examine the connected coverage problem in wireless sensor networks with the objective of providing *coverage* over a set of control points with minimum number of sensors. A point is covered if it is within the range of at least one sensor. In contrast to [9], our goal is to provide a minimum required SINR throughout the network which is a more desirable property compared to just covering the network area. Moreover it considers only a restricted form of sensors in which a sensors covers only a *single* directional beam. It has been shown that such a restriction for antennas results in a sub-optimal network performance [6]. In this paper, we consider a general antenna model and find the optimal wireless plan that satisfies a pre-specified performance constraint for all the network users.

The main contributions of this paper are as follows:

- We define WPP formally and establish its correspondence to NP-hard set-cover problem.
- We then consider three variations of the problem representing different antenna beam patterns, and provide *Integer Linear Program*s to find the optimal wireless plan for each variation.
- We propose a polynomial-time heuristic to solve WPP and through numerical results we demonstrate its efficiency.

The rest of the paper is organized as follows. Section II provides an overview of smart antennas and our system model. In Section III, WPP is formally defined and its complexity is discussed. Our optimization framework is formulated in Section IV. In Section V, a polynomial-time algorithm is presented to solve large-scale instances of the problem. Sample numerical results are provided in Section VI. Finally, Section VII concludes this paper.

II. System Model and Assumptions

A. Antenna Model

A K-element smart antenna consists of K omni-directional antenna elements with sophisticated signal processing algorithms capable of identifying signal's direction of arrival and calculating beamforming weights $\mathbf{w} = [w_1, w_2 \ldots, w_K]$ based on that direction. The objective is to estimate the direction that makes highest Signal-to-Noise-Ratio (SNR) at the receiver and determine amplitude and phase of each element so that the beam would be created towards that direction. Each w_i is a complex number of the form $w_i = |w_i| e^{-\jmath \phi_i}$, where $|w_i|$ and ϕ_i denote, respectively, the amplitude and phase of the signal generated by antenna element i.

There are two main types of smart antennas, namely adaptive and switched antennas. In the former, beamforming weights are computed based on receiver channel conditions, while in the latter, beam patterns are fixed and weights are pre-computed. Although switched smart antennas provide lower antenna gains for specific users, they do not need instantaneous channel feedback from the receivers. Thus, in this work, we consider switched antennas, which achieve considerable performance at a lower complexity. A K-element switched antenna can produce K distinct beams in different directions. Let $\mathcal{D} = \{d_1, d_2, \ldots, d_K\}$ denote the set of possible directions.

For consumer devices, typically, omni-directional antennas are used because of size limitations. Therefore, the system we consider in this paper consists of smart antenna APs and typical omni-directional clients. Similar to [1], we consider a two-phase TDMA-based MAC protocol to synchronize downlink and uplink traffic. The first phase is dedicated to directional transmissions from APs to clients, while the second phase is reserved for clients omni-directional transmissions. The majority of the traffic in enterprise WLANs tends to be downlink, therefore, our goal is to find a network coverage plan that meets downlink traffic requirements.

B. Network Model

The indoor area is discretized into a set of n possible user locations $\mathcal{L} = \{l_1, l_2, \ldots, l_n\}$ that must be covered by the wireless plan. This is a natural approximation of the environment (as users cannot be in any arbitrary location) that gives the problem a discrete structure rather than a continuous structure which is more difficult to formulate. This approximation could be made as accurate as needed by considering more possible user locations. There is also a set of m potential antenna locations denoted by $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$. Each of a_i's is a possible location for placing an antenna and no more than one antenna can be located at a point.

C. Communication Model

Let $P_i \leq \mathrm{P}_{\max}$ denote the transmission power of antenna a_i, *i.e.*, the antenna placed at location $a_i \in \mathcal{A}$. We assume that each antenna can choose a transmission power from a set of power levels $\mathcal{P} = \{p_1, p_2, \ldots, p_l\}$, where $p_i \leq \mathrm{P}_{\max}$ for $1 \leq i \leq l$.

Let P_{ij} denote the power received by location $l_j \in \mathcal{L}$ from antenna a_i. Two propagation models, namely the Protocol Model and the Physical Model [10], are widely used in the literature for modeling the effect of interference in wireless networks. In this paper, we consider the more complicated Physical Model as it provides a more accurate representation of the average behavior of received signal power in a wireless network.

In the Physical Model, a user at location l_j successfully receives information from its associated AP, a_i if the SINR of the a_i's transmission at l_j is beyond a threshold β_j, otherwise nothing can be received. Let γ_j denote the SINR at location l_j and $\mathcal{T} = \{t_1, t_2, \ldots, t_l\}$ denote the set of active transmissions other than a_i's transmission. Then γ_j is given by

$$\gamma_j = \frac{P_i / d_{ij}^{\alpha}}{\sum_{t_k \in \mathcal{T}} P_k / d_{kj}^{\alpha} + \eta_j} \tag{1}$$

where, d_{ij} is the distance between a_i and l_j, $\alpha \geq 2$ is the path-loss exponent, and η_j denotes the noise power at location l_j. For notational simplicity, define g_{ij} as $g_{ij} = 1/d_{ij}^{\alpha}$.

The bit-rate achievable by user located at l_j is a non-decreasing function of γ_j. For instance, using Shannon's formula, the maximum error-free rate λ_j achievable by user j is given by $\lambda_j = \log(1 + \gamma_j)$. Thus by using different thresholds γ_j across users, non-uniform traffic demand in the network could be addressed.

III. Wireless Planning Problem

In this section, we formalize our definition of a *wireless plan* and define the *wireless planning problem* that is considered in this paper.

Definition 1 (Wireless Plan)**.** *Let $\mathcal{U} = \mathcal{A} \times \mathcal{B}$ be the universe of possible beam pattern placements, where $\mathcal{A}$ denotes the set of potential antenna locations and $\mathcal{B}$ denotes the set of possible beam patterns that is defined as $\mathcal{B} = \{(b_1, ..., b_K) | b_i \in \mathcal{P}; \sum_{1 \leq i \leq K} b_i \leq P_{max}\}$. A nonempty set $\mathcal{W} \subseteq \mathcal{U}$* s.t

$\mathcal{W} = \{(a_i, b_{l_i}) | a_i = a_j \Rightarrow b_{l_i} = b_{l_j}\}$ *is called a wireless plan.*

According to definition of wireless plan, two antenna can not be placed at the same locations. All the wireless plans do not satisfy the coverage requirements of the network. The following definition specifies a plan by which the coverage condition is satisfied.

Definition 2 (Desirable Wireless Plan)**.** *Given a prespecified SINR threshold β_j for each location $l_j \in \mathcal{L}$, A wireless plan $\mathcal{W}^+$ is called desirable if $\forall l_j \in \mathcal{L}, \gamma_j > \beta_j$.*

Due to the cost associated with deployment and maintenance of a wireless network, the objective of network planning process is to minimize the number of antennas required to cover the entire network.

Definition 3 (Wireless Planning Problem)**.** *The objective of the wireless planning problem is to find a desirable wireless plan $\mathcal{W}^*$ that minimizes the number of required antennas. More formally, the wireless planning problem is defined as*

$$\mathcal{W}^* = \arg\min_{\mathcal{W}^+} \sum_{(a_i,.)\in\mathcal{W}^+} 1 \tag{2}$$

.

A. Computational Complexity

Wireless LAN planning problem has be shown to be NP-hard [8] [7]. Thus there is no hope of finding any polynomial time algorithm for it. Actually this problem is an extension of NP-hard set-cover problem [11]. WPP formulations that will be presented in the paper are all based on integer formulation of set-cover problem, thus in this section we present this problem and show that how a simpler form of WPP can be cast to this problem. Set-cover is defined as follows,

Definition 4 (Set-cover Problem)**.** *Given a universe $\mathcal{U}$, a collection of subsets of $\mathcal{U}$ denoted by $\mathcal{S} = \{S_1, \ldots, S_T\}$, and a cost function $c : S \rightarrow R^+$, find a minimum cost subcollection of $\mathcal{S}$ denoted by $\mathcal{C}$ that covers all elements of $\mathcal{U}$.*

Consider a simpler variation of WPP in which interference is ignored in computing SINR for all locations.We refer to this variation as WPP-S. We illustrate constructively the mapping of WPP-S to set-cover problem.

Here, the universe $\mathcal{U}$ is the set of locations to be covered $\mathcal{U} = \mathcal{L}$ and collection of subsets $\mathcal{S}$ is defined as $\mathcal{S} = \{\mathcal{S}_{im} \mid \mathcal{S}_{im}$ is the set of all locations $l_j \in \mathcal{L}$ that are covered by $(a_i, b_m) \in \mathcal{A} \times \mathcal{B}\}$. Let the cost of a set $\mathcal{S}_{im}$ be the price of installing an antenna at a_i. Considering equal cost for all the antennas, the objective of WPP-S is to find the minimum number of a_i's which covers the universe $\mathcal{L}$.

We say a point l_j is covered by $\mathcal{S}_{im}$ when it is within the area of one of the beams of b_m such that the transmitted power p on that beam, meets the requirement of SINR model. Since in WPP-S, we don't consider interference from other concurrent transmissions, equation (1) is reduced to

$$p \geq d_{ij}^{\alpha} \eta_j \beta_j \tag{3}$$

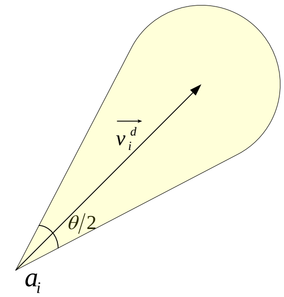

Fig. 1. Beam pattern of a directional antenna.

We model the antenna beam as a circular sector in which the relationship between the beam and the set of points it covers is determined by the Target In Sector (TIS) test [12]. This test states that a point l_j is covered by beam d of an antenna located at a_i if both of the following conditions are satisfied:

1) $d_{ij} \leq r$,
2) $\vec{v_i^d} \cdot \vec{d_{ij}} \geq d_{ij} * \cos(\frac{\theta}{2})$,

where $\vec{d_{ij}}$ is the vector from location a_i to l_j, $\vec{v_i^d}$ is a unit vector that denotes the beam direction, r is the transmission range (obtained from 3) and θ is the central angel of the beam (see Fig. 1).

The first condition in TIS checks if the point l_j is within the transmission range of the antenna placed at a_i. The second condition checks if the vector $\vec{d_{ij}}$ is within the central angle of the antenna beam by computing the inner product of $\vec{v_i^d}$ and $\vec{d_{ij}}$. The equality holds when point l_j is along one of the two edges of the transmission sector of the antenna beam.

A brute force solution is to consider all subsets of $\mathcal{S}$ in increasing order of cardinality, check the coverage condition 1, and output the first subset that satisfies it. However, there is an exponential number of subsets which results in worst-case exponential running time. In the following sections we formulate the problem as a integer linear program and present a polynomial-time heuristic to solve it.

IV. Wireless Planning Formulation

In this section, we derive integer linear programs for several instances of WPP. In derivation of these formulations we basically extend integer formation of set-cover problem 4 by adding constraints of WPP to it.

$$\begin{aligned} \min \quad & \sum_{S_i \in \mathcal{S}} w_i x_{S_i} & \\ \text{s. t.} \quad & \sum_{S_i:\, u \in S_i} x_{S_i} \geq 1 & \forall u \in \mathcal{U} \\ & x_{S_i} \in \{0, 1\} & \forall S_i \in \mathcal{S}\,. \end{aligned} \tag{4}$$

Where w_i is the weight assigned to set S_i. The constraint indicates coverage of every member u of the universal set $\mathcal{U}$ by at least one covering subset. The objective function indicates the goal to minimize the cost of the cover.

The SINR constraint described in section II is a nonlinear function, hence to simplify the formulation, we pre-compute

g_{ij} for every pair $(a_i, l_j) \in \mathcal{A} \times \mathcal{L}$. This computation takes $O(|\mathcal{A}||\mathcal{L}|)$ time, where $|\mathcal{A}|$ and $|\mathcal{L}|$ are the cardinalities of the sets $\mathcal{A}$ and $\mathcal{L}$ respectively.

We also need to know the set of points that is covered by each antenna beam-pattern pair. As we have already mentioned we use TIS test to find this set. Although the power radiated from an antenna in the form of an electromagnetic signal decays as the signal travels in the environment but does not disappear completely over finite distances of an indoor environment. Thus, we assume that within the central angle of the antenna beam the antenna coverage range is unbounded. Obviously, the boundaries of the indoor environment(set $\mathcal{L}$) put a limit on the cardinality of the covered set by each beam. Matrix $C[c_{j,im}]_{|\mathcal{L}|,|\mathcal{A}||\mathcal{B}|}$ represents coverage relationship between all $(l_j; a_i, b_m)$ pairs such that

$$c^d_{ji} = \begin{cases} 1, & \text{If location } l_j \text{ is under the coverage of antenna } a_i \\ & \text{when it radiates based on beam pattern } m, \\ 0, & \text{otherwise.} \end{cases}$$

Computing matrix C takes $O(|\mathcal{A}||\mathcal{L}||\mathcal{B}|)$ time.

We consider three different beam pattern types, namely *Constant Power Single-Beam*, *Dynamic Power Single-Beam*, *Dynamic Power Multi-Beam*. These antenna patterns differ in the number of beams that an antenna can radiate and the set of available power levels for each beam.

A. Constant Power Single-Beam System

In this system, antennas radiate a single-beam with maximum power $\mathrm{P}_{\max}$. There are $\mathcal{D}|$ beam patterns totally. Thus, we only need to find the optimal antenna locations and orientations but not the power levels. To formulate WPP in this case, we introduce a set $\mathcal{X} = \{x^d_i | a_i \in \mathcal{A}, d \in \mathcal{D}\}$ of binary decision variables to represent the decision space of the problem where x^d_i is defined as follows

$$x^d_i = \begin{cases} 1, & \text{If an antenna is placed at location } a_i \text{ and} \\ & \text{directed towards direction } d, \\ 0, & \text{otherwise.} \end{cases}$$

Another set of decision variables $\mathcal{H} = \{h^d_{ji} | l_j \in \mathcal{L}, a_i \in \mathcal{A}, d \in \mathcal{D}\}$ is defined to capture the associations of $l_j \in \mathcal{L}$ to $x^d_i \in \mathcal{X}$ as follows

$$h^d_{ji} = \begin{cases} 1 & \text{If location } l_j \text{ is associated to antenna beam } x^d_i \\ 0 & \text{otherwise.} \end{cases}$$

Using the above definitions, WPP can be represented as the following *Integer Linear Program*:

$$\min \quad \sum_{i,d} x^d_i \tag{5a}$$

s. t.

$$\sum_{d} x^d_i \le 1, \qquad \forall a_i \in \mathcal{A} \tag{5b}$$

$$\frac{\sum_{i,d} h^d_{ji} x^d_i P_{ij}}{\sum_{i,d}(1 - h^d_{ji}) c^d_{ji} x_{id} P_{ij} + \eta_j} \ge \gamma_j, \quad \forall l_j \in \mathcal{L} \tag{5c}$$

$$\sum_{i,d} h^d_{ji} \le 1, \qquad \forall l_j \in \mathcal{L} \tag{5d}$$

$$h^d_{ji} \le x^d_i, \qquad \forall (l_j, a_i, d), l_j \in \mathcal{L}, a_i \in \mathcal{A}, d \in \mathcal{D} \tag{5e}$$

$$h^d_{ji} \le c^d_{ji}, \qquad \forall (l_j, a_i, d), l_j \in \mathcal{L}, a_i \in \mathcal{A}, d \in \mathcal{D} \tag{5f}$$

The objective function (5a) captures our desire to minimize the number of antennas used in the plan. (5b) enforces existence of at most one antenna at each AP location; by constraint (5c), coverage of each location l_j according to SINR model is guaranteed; the numerator in the constraint takes the received power from beam d of antenna a_i if it is installed and l_j is associated to it. The denominator takes the sum of received powers from all existing antennas which cover l_j and l_j is not associated to them plus η_j. (5d) indicates that each location l_j should be associated to at most one antenna beam. Constraints (5e) and (5f) denote that when l_j is associated to beam d of the antenna a_i, such beam must exist and cover l_j.

Due to inclusion of $h^d_{ji} x^d_i$ term, the constraint (5c) is quadratic. However, because of constraints (5e) and (5f) and the fact that h^d_{ji}, x^d_i, and c^d_{jk} all take $\{0, 1\}$ values, we have $h^d_{ji} x^d_i = h^d_{ji}$ and $h^d_{ji} c^d_{ji} = h^d_{ji}$. Thus, this constraint can be represented as follows:

$$\frac{\sum_{i,d} h^d_{ji} P_{ij}}{\sum_{i,d} c^d_{ji} x^d_i P_{ij} - \sum_{i,d} h^d_{ji} P_{ij} + \eta_j} \ge \gamma_j \tag{6}$$

or, equally as the following linear constraint:

$$\sum_{i,d} h^d_{ji} P_{ij} - \gamma_j (\sum_{i,d} c^d_{ji} x^d_i P_{ij}) + \gamma_j (\sum_{i,d} h^d_{ji} P_{ij}) - \gamma_j \eta_j \ge 0 \tag{7}$$

B. Dynamic Power Single-Beam System

With fixed power assignment, the only available options to cover all the points in the area are installing new antennas or changing the locations and directions of current available antennas. Nevertheless, based on the SINR capture model, it is possible to change transmission powers of deployed antennas and cover new points. Since the transmission power of an antenna can take any value $p_l \in \mathcal{P}$, there exist $|\mathcal{P}||\mathcal{D}|$ beam patterns in this case.

To include variable power assignment in our optimization framework, decision variables x^d_i and h^d_{ji} are redefined as

$$x^d_i[l] = \begin{cases} 1, & \text{If an antenna is placed at } a_i \text{ , directed towards} \\ & \text{direction } d \text{ and transmit at power } p_l, \\ 0, & \text{otherwise.} \end{cases}$$

and

$$h^d_{ji}[l] = \begin{cases} 1, & \text{If location } l_j \text{ is associated to antenna } x^d_i[l], \\ 0, & \text{otherwise.} \end{cases}$$

These variables are substituted in 5a,5b,5c,5d, 5e,5f, and P_{ij} is also adjusted to obtain the formulation of dynamic power single-beam system.

C. Dynamic Power Multi-Beam System

Smart antennas can radiate multi-beam beam patterns by combining multiple beams in different directions and adjusting amplitude and phase of them. In this case, the supplied power is distributed among active beams either uniformly or non-uniformly, but the sum of allocated powers is bounded above to $\mathrm{P}_{\max}$. If the difference of each two consecutive power levels is the same $p_{i+1} - p_i = P_{max}/l, \forall 1 \leq i \leq |\mathcal{P}|-1$, the number of these patterns is equal to $\binom{|\mathcal{P}|+|\mathcal{D}|}{|\mathcal{D}|}$. Following the method of previous subsections, new binary variable are defined. We define x_i^b as the binary variable which takes value 1 when there is an antenna at location a_i radiating beam pattern b. Also binary variable h_{ji}^b captures association of location l_j to x_i^b. By substituting these variables in 5a,5b,5c,5d, 5e,5f, we obtain the formulation for a dynamic power multi-beam system.

D. Discussion

The above formulations all can be solved using integer programming algorithms like branch and cut implemented in various optimization softwares. Branch and cut employs a linear programming algorithm such as Simplex to find the optimal solution to integrability-relaxed version of the problem. Since the solution may not be integral, through techniques like cutting plane and branching suitable inequality constraints are found and augmented to the problem to forbid the same fractional results happen again. These steps are done successively until an optimal integer solution is achieved. This approach may need exponential number of iterations to explore the entire integer domain of the problem. In addition, even medium-size instances of WPP results a large number variables and constraints. Thus, finding the optimum solution for large instances of WPP using integer programming solvers may become impractical. However, even a suboptimal solution might be much better than ad hoc planning and be close to optimal one. In the following section we present our polynomial-time heuristic to find such a solution.

V. Greedy Algorithm

Branch and cut considers all possible pairs of (antenna, beam patterns) and prunes the space successively to find the optimal solution. Based on set-cover heuristic [11], we present a heuristic in which WPP is solved for one antenna at a time, choosing appropriate place and beam pattern for it. This leads to a greedy approach in which we make a locally optimum decision at the moment with hope that sequence of locally optimum decisions achieves globally optimal solution. The algorithm is called GreedySelect and shown in algorithm 1. We show the list of selected (antenna,beam patterns) as $\mathcal{S}$ and covered locations by $\mathcal{C}$. At first both sets are empty. In each iteration, the pair of antenna location and beam pattern that covers maximum uncovered user locations is selected and added to $\mathcal{S}$. This process continues until all user locations are covered or addition of new antennas isn't possible or doesn't improve coverage. When an antenna enters $\mathcal{S}$ never leaves it, however introduction of new antennas may remove some locations from $\mathcal{C}$. Since, there are at most $|\mathcal{A}|$ antennas available, the number of iterations is $O(|\mathcal{A}|)$. Finding and updating information at each iteration takes $O(|\mathcal{A}||\mathcal{L}|)$, so the time complexity of GreedySelect is $O(|\mathcal{A}|^2|\mathcal{L}|)$.

Algorithm 1: GreedySelect Algorithm

```
Input: A, L
Output: S, C
begin
    S ⟵ ∅;
    C ⟵ ∅;
    while |S| < |A| and |C| < |L| do
        Cnum ← |C|;
        foreach (a_i, b_k) ∉ S do
            S ← S ∪ (a_i, b_k);
            foreach l_j ∈ L do
                Compute γ_j;
            Covered ← Σ_j 1_{γ_j > β_j};
            if Covered > Cnum then
                (besta, bestb) ← (a_i, b_k);
                Cnum ← Covered;
            S ← S − (a_i, b_k);
        if Cnum > Σ_j 1_{γ_j > β_j} then
            S ← S ∪ (besta, bestb);
            C ← {j | γ_j > β_j};
```

VI. Numerical Results

In this section, we provide numerical results to compare different antenna placement scenarios.

A. Network Setup

User locations are distributed according to 2D Poisson distribution with parameter $\lambda = 1$ in a square area. A user location is selected with probability $p = 0.7$ as a possible antenna location. K is set to 4, thus there are four possible beam directions each with the central angle of $\frac{\pi}{2}$. The set of power levels $\mathcal{P}$ includes four values of $1, 2, 3, 4$. The path loss exponent α is assumed to be 2. For each location l_j, SINR threshold β_j, and noise η_j are set to 1. In the results, Optimal refers to the results of integer formulations and Greedy refers to the results of GreedySelect algorithm. We use Fixed, Variable, and Multi as abbreviations of constant power single-beam system, dynamic power single-beam system, and dynamic power multi-beam system respectively.

B. Results

we increase area of the network from 9 to 36. λ remains fixed, so the average number of user locations will be increased from 9 to 36. For each network size, we average the results of Greedy and Optimal over 10 runs.

The optimal number of antennas required to cover all locations is demonstrated in figure 2. These results compare different beam pattern types. As observed, dynamic power multi-beam pattern constantly outperforms other types of beam patterns. We also measured the results of greedy algorithm for

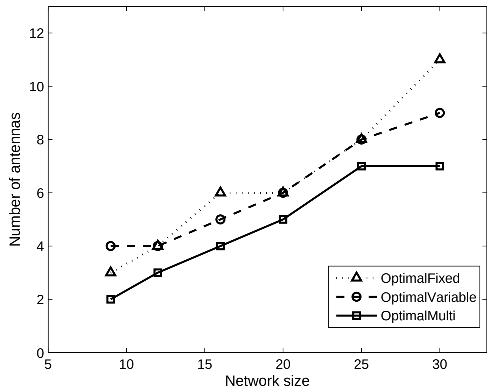

Fig. 2. Optimal number of antennas to cover the network

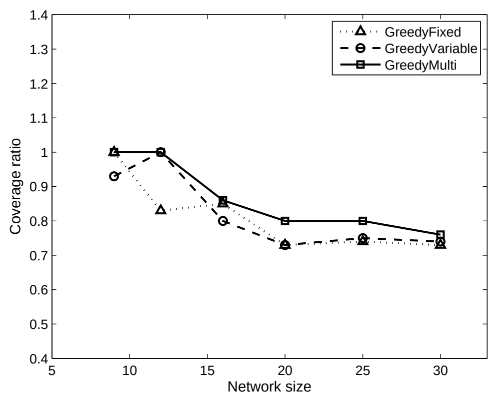

Fig. 3. Coverage ratio of GreedySelect algorithm

different beam patterns to compare them with the associated optimal ones. Greedy algorithm may fail to find a total coverage plan when it is available. We define coverage ratio as the ratio of the number of locations covered by the greedy algorithm to the number of locations. The coverage ratio achieved by greedy algorithm is shown in figure 3. For small network sizes where maximum distance between two locations in network is comparable to antenna transmission radius, we observe total coverage. However for larger network sizes when there isn't such a relationship, GreedySelect almost achieves 75% coverage. As apparent from the plot, we expect this ratio continues to hold. This plot suggests that for a random distribution of nodes GreedySelect finds a plan with constant coverage ratio(near 1).

When GreedySelect algorithm covers all locations, it typically needs more antennas than the optimal one. We define antenna ratio as the ratio of the number of antennas employed in GreedySelect to the optimal case. It is another criterion which together with coverage ratio show the behavior of GreedySelect. The antenna ratio is depicted in figure 4. As apparent for larger network sizes, antenna ratio is almost 0.8. All in all, antenna ratio of 0.8 and coverage ratio of 0.75 confirm the efficiency of the greedy algorithm.

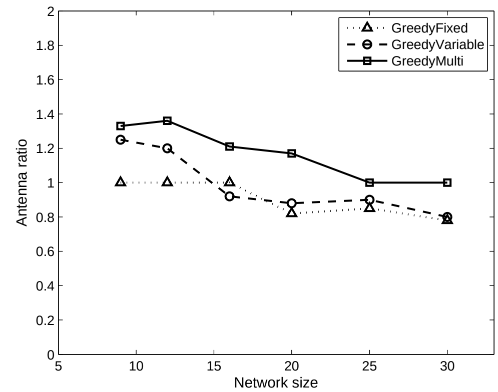

Fig. 4. Antenna ratio of GreedySelect algorithm

VII. Conclusion

The problem of indoor wireless planning using *smart antennas* is investigated in this study. We formulate *single-beam* and *multi-beam* cases of WPP as integer linear programs. Due to complexity of finding the optimal solution, we also present a greedy heuristic. Although numerical results confirm efficiency of greedy algorithm, optimal solutions are always desirable. Thus in the future, we intend to apply integer programming decomposition methods to make finding optimal solution more tractable. Considering fairness issues in the planing problem is another interesting area for future research.

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