

CPSC 031 — Mathematics Review for CPSC 413

Warmup Problems for Mathematical Induction

September, 2000

These problems can be solved using mathematical induction. They are a bit simpler than the problems in Exercise #1, so you can start with these if you need practice in using mathematical induction, and find the problems in Exercise #1 to be too challenging to start with.

Note that you might also be able to solve some of these problems, in a different way, without using mathematical induction at all! However, the purpose of this exercise is to give you practice using mathematical induction, so you should look for a way to *use* mathematical induction when you answer these questions.

1. Prove that $n^2 - n$ is an even number, for every integer $n \geq 0$.
2. Prove that $n^3 - n$ is divisible by 6, for every integer $n \geq 0$. You may use the fact that you proved in the first question, without proving it again.

3. Prove that

$$\sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

for every integer $n \geq 0$.

4. Prove that

$$\sum_{i=0}^n i^3 = \frac{n^2(n+1)^2}{4}$$

for every integer $n \geq 0$.

5. Prove that every binary tree with n edges has exactly $n + 1$ nodes, for every integer $n \geq 0$.
6. Prove that every (nonempty) binary tree with depth d has at least $d + 1$ nodes and at most $2^{d+1} - 1$ nodes, for every integer $d \geq 0$.