CPSC 031 — Mathematics Review for CPSC 413

Exercise #3 — Integration

September, 1999

Please try these exercises before the 6pm lecture on September 10.

Compute each of the following integrals, assuming a and b are positive real numbers such that $b > a > 1$.

1. $\displaystyle\int_a^b \frac{x^3 - 1}{x - 1}\, dx$
2. $\displaystyle\int_a^b (2 - \sqrt{x})\, dx$
3. $\displaystyle\int_a^b e^x\, dx$
4. $\displaystyle\int_a^b x + \tfrac{1}{x}\, dx$
5. $\displaystyle\int_a^b \frac{1}{x - 1}\, dx$
6. $\displaystyle\int_a^b \frac{(\ln x)^3}{x}\, dx$
7. $\displaystyle\int_a^b x^2 e^x\, dx$
8. $\displaystyle\int_a^b \tfrac{1}{(x-1)(x+1)}\, dx$ (This one is challenging!)

Hints

Exercise #1: The integral can be calculated by performing a simple algebraic substitution: Note that $a > 1$ and that $x - 1$ is a factor of the numerator of the expression to be integrated.

Exercise #5: You might calculate this integral by using an application of "integration by substitution" resembling one of the examples given in the Fall, 1998 online notes for this topic.

Exercise #6: Consider "integration by substitution" for this integral as well. One of the functions involved will be the natural logarithm, $\ln x$.

Exercise #7: Consider "integration by parts."

Exercise #8: You'll probably need to manipulate the expression to be integrated, in order to compute the integral. Fractions of polynomials can be decomposed in a particular way, and there exist *constants* c_1 and c_2 such that

$$\frac{1}{(x-1)(x+1)} = \frac{c_1}{x+1} + \frac{c_2}{x-1}.$$

Since these two expressions are equal their indefinite (and definite) integrals are as well. However, you'll probably find it easier to integrate the expression on the right hand side (once you've solved for c_1 and c_2) than the one on the left.