CPSC 031 — Mathematics Review for CPSC 413

Exercise #2 — Limits and Derivatives

September, 1999

Please try these exercises before the 6pm lecture on September 9.

1. Compute each of the following limits or explain why it does not exist.

   (a) $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

   (b) $\lim_{x \to +\infty} \dfrac{x^2 + 2x + 1}{3x^2 + 5}$

   (c) $\lim_{x \to 0} \dfrac{x}{\cos(x) - 1}$

   (d) $\lim_{x \to 0} \dfrac{\sin(4x)}{\sin(3x)}$

2. Compute the derivative (with respect to x) of each of the following functions.

   (a) $f(x) = 3x^2 + 2x + 1$

   (b) $f(x) = x \ln x$

   (c) $f(x) = x / \ln x$

   (d) $f(x) = e^{x^2 \ln x}$

3. Derive the Quotient Rule (as given below), starting with the definition of a derivative, and assuming that the functions f and g are both differentiable at a and that $g(a) \neq 0$.

   Quotient Rule: If $h(x) = \frac{f(x)}{g(x)}$ then

   $$h'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{(g(a))^2}.$$

4. Prove that

   $$\lim_{x \to +\infty} \frac{(\ln x)^n}{x} = 0$$

   for every natural number $n \geq 1$.